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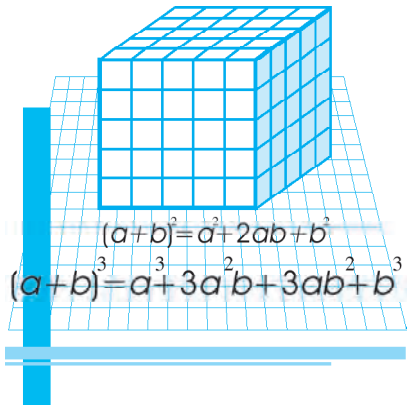
ALGEBRA

**Umumiy o‘rta ta’lim maktablarining
7- sinfi uchun darslik**

Qayta ishlangan va to‘ldirilgan
5- nashri

*O‘zbekiston Respublikasi Xalq ta’limi
vazirligi tasdiqlagan*



„O‘QITUVCHI“ NASHRIYOT-MATBAA IJODIY UYI
TOSHKENT — 2017

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Aziz o‘quvchilarim!

Ona yurtimiz mustaqil O‘zbekiston jahon ilm-u faniga, madaniyatiga yuzlab buyuk olimlarni, shoirlarni, davlat arboblarini, musavvirlarni yetishtirib bergan. Bilingki, siz ularning ezgu ishlari davomchisisiz!

Yoshlik bilim olish davridir.

Allomalar aytadi: „Yoshlikda olingan bilim toshga bitilgan yozuv kabi o‘chmasdir“. Algebrani, umuman, matematikani o‘rganish qunt va izchillikni, ko‘plab masala va misollarni tushunib, idrok qilib yechishni talab etadi. Meni yaxshi o‘rganib olsangiz, sizga umrbod do‘st bo‘lib qolaman!

Xulq-u odobingiz barkamol, ilmingiz ziyoda bo‘lishini istab

„*Algebra*“ darsligingiz.

Darslikdagi shartli belgilar:

 — asosiy qoidalar va xossalar.

○ — matematik tasdiqni asoslash yoki formulani keltirib chiqarish boshlandi.

● — asoslash yoki formulani keltirib chiqarish tugadi.

△ — masalani yechish boshlandi.

▲ — masalani yechish tugadi.

№ — qiziqarli masalalar.

25, 42, ... — murakkabroq masala.

 — sinov mashqlari.

 — asosiy material bo‘yicha bilimni tekshirish uchun mustaqil ish.

 — tarixiy ma’lumotlar.

Darslikdagi pul muomalasiga doir mashqlarda narxlar shartli qabul qilingan.

Respublika maqsadli kitob jamg‘armasi mablag‘lari hisobidan chop etildi.

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5—6- SINFLARDA O‘RGANILGAN MAVZULARNI TAKRORLASH

Aziz o‘quvchi! Siz 5—6- sinflarda natural sonlar, oddiy va o‘nli kasrlar, ratsional sonlar ustida to‘rt amalga doir misol va masalalarni yechgansiz. 5—6- sinflarda matematikadan olgan bilimlaringizni yodga solish maqsadida Sizga bir nechta mashqlar taklif etamiz.

1. „Xalq bilan muloqot va inson manfaatlari“ yilida qurilgan zamonaviy uylar shahrimizga yanada ko‘rk bag‘ishladi. Yangi qurilgan ko‘p qavatli uylardan birining xonadonlari 1, 2, 3, ..., 99, 100 sonlari bilan nomerlangan. Raqamlari yig‘indisi o‘zaro teng bo‘lgan xonadonlar nechtadan? Natijalarni jadvalda va diagrammada aks ettiring.

2. Bir fermadagi sigirlar soni 2- fermadagiga qaraganda 12 % kam. Ammo 1- fermaning har bir sigiri 2- fermaning har bir sigiriga qaraganda 7,5 % ko‘p sut beradi. Qaysi ferma va necha foiz ko‘p sut oladi?

3. 300 kg g‘alla ma’lum muddat quritilgach, uning massasi 20 kg ga kamaydi, namligi esa 10 % ni tashkil qildi. Dastlab g‘allaning namligi necha foiz edi?

4. Tenglamani yeching:

1) $5x+48:4=20:10+2\cdot10$;

2) $7x+32:2=(72+18):3$;

3) $4\frac{1}{2}x+3\frac{3}{10}\cdot5=7\frac{6}{13}+18\frac{7}{13}$;

4) $6\frac{1}{2}x+3\frac{1}{2}\cdot3=11\frac{4}{17}+5\frac{13}{17}$.

5. Ahmad velosipedda soatiga 10,8 km tezlik bilan 1 soat-u 15 minut yo‘l yurdi. So‘ngra soatiga 12,8 km tezlik bilan 2,5 soat yo‘l yurdi. Ahmad jami necha kilometr yo‘l yurgan?

6. To‘g‘ri to‘rtburchakning bo‘yi 8 sm ga teng. Eni bo‘yidan 1,5 sm qisqa. To‘g‘ri to‘rtburchakning yuzini toping.

7. To‘g‘ri to‘rtburchakning yuzi 20,25 dm^2 ga, eni 3,24 dm ga teng. Shu to‘g‘ri to‘rtburchakning perimetrini toping.

8. Avtomobil 100 km masofaga 5 *l* benzin sarflaydi. Bu avtomobil: 50 km; 60 km; 70 km; 80 km; 120 km; 250 km; 360 km yo‘lga qancha benzin sarflaydi?

9. Sayyoh yo‘lning $\frac{2}{7}$ qismini o‘tdi. Hisoblab ko‘rsa, yo‘lning yarmiga yetishi uchun yana 9 km yurishi kerak ekan. Sayyoh jami necha kilometr yo‘l yurishni mo‘ljallagan?

10. Bir avtomobil 100 km masofaga 8 *l,* ikkinchi avtomobil esa shuncha masofaga 10 *l* benzin sarflaydi. Agar har bir avtomobil bakida 32 *l* dan benzin bo‘lsa, bu yonilg‘i ular uchun necha kilometr yo‘lga yetadi?

11. 1) Matoning narxi 20 % pasaytirildi. Ma’lum vaqtdan so‘ng, yangi narx ham 25 % pasaytirildi. Matoning narxi jami necha foiz kamaygan?
2) Gazlamaning narxi 20 % ortdi. Ma’lum vaqtdan so‘ng, yangi narx ham 25 % ortdi. Gazlamaning narxi jami necha foiz ortdi?

12. Bug‘doyning namligi 23 % edi. U quritilgach, namligi 12 % ga tushdi. Bug‘doyning massasi necha foizga kamaydi?

13. Tadbirkor 1- va 2- nav mollarni sotib, jami 54 000 so‘m foyda qildi. 1- nav molning narxi 120 000 so‘m edi, tadbirkor uni 15 % foydasiga sotdi. 2- nav moldan 20 % foyda ko‘rdi. 2- nav molning narxi necha so‘m? Ikkala nav molni sotib, tadbirkor necha foiz foyda ko‘rgan?

14. To‘g‘ri to‘rtburchak asosining uzunligi 20 %, balandligi 25 % orttirilsa, uning yuzi necha foiz ortadi?

15. To‘g‘ri to‘rtburchak asosining uzunligi 10 %, balandligi 20 % kamaytirilsa, uning yuzi necha foiz kamayadi?

16. Amallarni bajaring:

1) $(-120):\big((-8)\cdot(-3)+12:(-3)\big)-(-48):(-16)$;

2) $(-75)\cdot 4-204:(-3)+(-210):(-7)$;

3) $(-20,25):(-3,6)+90,72:(-4,5)-7,5\cdot 3,2$;

4) $5\frac{5}{19}\cdot(-0,95)+2\frac{16}{17}\cdot(-0,34)-8\frac{4}{7}:2\frac{1}{7}$.

17. Tenglamani yeching:

1) $3x+2x=17+(-27)$;

2) $6x-7x=3,5\cdot(-1)+4$;

3) $1,3x-3,5x=11\cdot(-0,5)$;

4) $4x-2\frac{1}{3}x=3\frac{1}{3}:(-2)$.

18. 5 ta sonning o‘rta arifmetigi 18,4 ga teng. Bu sonlarga yana bitta son qo‘shib, o‘rta arifmetik qiymat hisoblangan edi, u 20 ga teng chiqdi. Qo‘shilgan sonni toping.

19. Karim ota 90 yoshda. Uning nabiralarining o‘rtacha yoshi 20 da. Nabiralar yoshlariga Karim ota yoshini ham qo‘shib, o‘rta arifmetik qiymat hisoblangan edi, u 22 ga teng chiqdi. Karim otaning nechta nabirasi bor?

20. Avtomobil 72 km/soat tezlik bilan 3,5 soat, 60 km/soat tezlik bilan 2,5 soat yurdi. Avtomobil jami necha kilometr yo‘l yurgan? Bu masofani u qanday o‘rtacha tezlikda bosib o‘tdi?

21. Proporsiyaning noma’lum hadini toping:

1) $3,5:x=2,4:4,8$;

2) $x:2\frac{1}{3}=9,2:2,3$;

3) $7,2:2,4=x:4\frac{1}{3}$;

4) $4\frac{2}{7}:2\frac{1}{7}=3,2:x$.

I BOB
ALGEBRAIK IFODALAR

1- §. Sonli ifodalar

Algebra so‘zi buyuk o‘zbek matematigi va astronomi, vatandoshimiz Abu Abdulloh Muhammad ibn Muso al-Xorazmiyning „Kitob al-muxtasar fi hisob al-jabr val-muqobala“ („Al-jabr val-muqobala“) asaridagi *al-jabr* (lotinchasiga *algebra*) so‘zidan olingan. Bu asarda al-Xorazmiy dunyoda birinchi marta algebra fanini izchillik bilan bayon qilgan.

Algebraning asosiy masalasi algebraik ifodalar ustida matematik amallarni o‘rganishdir. Algebraik ifodalarning eng soddа ko‘rinishi bo‘lgan sonli ifodalar 5—6- sinf matematika kurslarida qaralgan edi.

Sonli ifoda sonlardan tuzilib, amallar belgilari bilan birlashtirilgan yozuv ekanligini eslatib o‘tamiz. Masalan, $2\cdot3+7$;

$10:2-3$; $\frac{4\cdot0,5+3}{5}$; $\frac{1}{3}-\frac{1}{2}$ yozuvlar sonli ifodalardir.

Sonli ifodaning qiymati *deb, shu sonli ifodada ko‘rsatilgan amallarni bajarish natijasida hosil bo‘lgan songa aytiladi.*

Masalan, $2\cdot3+7$ sonli ifodaning qiymati 13 soni, $\frac{1}{3}-\frac{1}{2}$ sonli ifodaning qiymati $-\frac{1}{6}$ sonidir.

Sonli ifoda bitta sondan iborat bo‘lishi ham mumkin. Uning qiymati shu sonning o‘zi bo‘ladi.

Ba’zan sonli ifodada sonlar va amallar belgilaridan tashqari amallarning ma’lum tartibda bajarilishini ko‘rsatuvchi qavslardan foydalaniladi. Masalan, $(2,5+3,5)\cdot2,1$ sonli ifodaning qiymatini hisoblashda avval qavs ichidagi qo‘shish amali, keyin ko‘paytirish amali bajariladi.

Abu Abdulloh Muhammad ibn Muso al-Xorazmiy (783—850) — buyuk o‘zbek matematigi va astronomi.

$(2,5 + 3,5) \cdot 2,1$ ifodaning qiymatini hisoblab, 12,6 sonini hosil qilamiz. Shuning uchun $(2,5 + 3,5) \cdot 2,1 = 12,6$ tenglikni yozish mumkin.

„=“ belgisi bilan birlashtirilgan ikkita sonli ifoda sonli tenglikni tashkil qiladi.

Agar tenglikning chap va o‘ng qismlarining qiymatlari bir xil son bo‘lsa, u holda tenglik ***to‘g‘ri tenglik*** *deyiladi.*

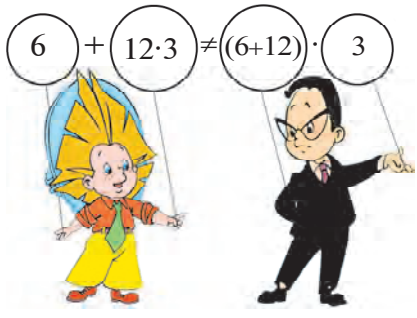
Masalan, $\frac{15-1}{2} = 8-1$ to‘g‘ri tenglik, chunki uning ikkala qismining ham qiymati 7 soniga teng.

Sonli ifodalar va sonli tengliklardan, hisoblashlar bilan bir qatorda, sonlarning xossalarini yozishda ham foydalaniladi.

Masalan, $\frac{3}{4} = \frac{6}{8}$ tenglik kasrlarning asosiy xossasini, $35 + 21 = 21 + 35$ tenglik esa qo‘shishning o‘rin almashtirish qonunini ifodalaydi.

Endi $6 + 12 \cdot 3$ sonli ifodani qaraylik. $6 + 12 \cdot 3 = 6 + 36 = 42$ dan iborat bo‘lgan to‘g‘ri natija amallarni qabul qilingan bajarish tartibiga rioya qilingan holdagina hosil bo‘ladi.

Agar qabul qilingan hisoblash tartibi buzilsa va avval 6 ga 12 ni qo‘shib, so‘ngra hosil bo‘lgan yig‘indi 3 ga ko‘paytirilsa, u holda 54 dan iborat noto‘g‘ri natija hosil qilinadi. Bu natija dastlabki ifoda $(6 + 12) \cdot 3$ kabi yozilsa, to‘g‘ri bo‘lar edi.



Demak, hisoblashning to‘g‘riligi sonli ifodadagi amallarning bajarilish tartibiga bog‘liq ekan.

Sonlar ustida amallarning bajarilish tartibi algebraik ifodalarning son qiymatlarini topishga oid masalalarni bajarishda ham saqlanib qoladi.

Qo‘shish va ayirish *birinchi bosqich amallar*, ko‘paytirish va bo‘lish esa *ikkinchi bosqich amallar* deyilishini eslatib o‘tamiz. Kvadrat va kubga ko‘tarish *uchunchi bosqich amallar* deyiladi.

Sonli ifodaning son qiymatini topishda amallar bajarilishining quyidagi tartibi qabul qilingan:

1) Agar ifodada qavslar bo‘lmasa, u holda avval uchinchi bosqich amallar, keyin ikkinchi bosqich amallar va, nihoyat, birinchi bosqich amallar bajariladi, shu bilan birga, bir xil bosqich amallar ular qanday tartibda yozilgan bo‘lsa, xuddi shu tartibda bajariladi.

Masalan,

$$3\cdot 5^2\cdot 4-5\cdot 4+7=3\cdot 25\cdot 4-5\cdot 4+7=300-20+7=280+7=287.$$

2) Agar ifodada qavslar bo‘lsa, u holda avval qavslar ichidagi sonlar ustida barcha amallar, so‘ngra esa qolgan barcha amallar bajariladi, bunda qavs ichidagi va undan tashqaridagi barcha amallar 1- bandda ko‘rsatilgan tartibda bajariladi.

Masalan,

$$(2^3\cdot 4-5)\cdot 6+(2+2\cdot 4)=(8\cdot 4-5)\cdot 6+(2+2\cdot 4)=$$

$$=(32-5)\cdot 6+(2+8)=27\cdot 6+10=162+10=172.$$

3) Agar kasr ko‘rinishidagi ifodaning qiymati hisoblanadigan bo‘lsa, u holda kasrning surati va maxrajidagi amallar bajariladi, so‘ngra birinchi natija ikkinchisiga bo‘linadi.

Masalan,

$$\frac{2\cdot 3^3-3\cdot 5}{3+5^2}=\frac{2\cdot 27-3\cdot 5}{3+25}=\frac{54-15}{28}=\frac{39}{28}=1\frac{11}{28}.$$

4) Agar ifodada qavslar ichida boshqa qavslar bo‘lsa, u holda avval eng ichkaridagi qavslar ichidagi amallar bajariladi.

Masalan,

$$2\cdot(8-(5^2-4))=2\cdot(8-(25-4))=2\cdot(8-21)=2\cdot(-13)=-26.$$

Mashqlar

1. Amallarni bajaring:

1) $2,17+(3,2-0,17)$;

2) $9,49-(1,5+0,99)$;

3) $13\frac{7}{9}-\left(2,64+2\frac{7}{9}\right)$;

4) $6\frac{7}{8}-\left(3,14-2\frac{1}{8}\right)$.

2. Sonli ifodaning qiymatini toping:

1) $\left(\frac{1}{2}+\frac{1}{3}\right)\cdot\left(\frac{1}{5}-\frac{1}{4}\right)$;

2) $\left(\frac{2}{7}-\frac{3}{4}\right)\cdot\left(\frac{2}{13}-\frac{1}{2}\right)$;

3) $\left(0,3-\frac{1}{20}\right):\left(\frac{3}{4}-1,25\right)$;

4) $\left(2,7-\frac{1}{5}\right):\left(\frac{1}{2}+4,5\right)$.

3. Qiymati: 1) 8; 2) 0; 3) 1; 4) -14 ga teng bir nechta sonli ifoda yozing.

4. Tenglik to‘g‘rimi:

1) $\frac{12,5-4,1}{4}=1,7+0,4$;

2) $\frac{0,75-0,15}{2}=0,15+0,25$;

3) $\frac{2,13+4,33}{7,58-4,35}=1\frac{5}{12}+\frac{1}{3}+\frac{1}{4}$;

4) $\frac{8,92-6,61}{5,38-1,55}=2\frac{1}{9}-\frac{1}{2}-\frac{1}{3}$?

Sonli tenglik shaklida yozing **(5—6):**

5. 1) $\frac{1}{3}$ va $\frac{1}{5}$ sonlari yig‘indisi $\frac{2}{3}$ va $\frac{2}{15}$ sonlari ayirmasiga teng;

2) 40 va 0,03 sonlari ko‘paytmasi 6 sonining 5 ga bo‘linmasiga teng.

6. 1) 10 va -2 sonlari ayirmasining ikkilangani shu sonlar yig‘indisidan uch marta katta;

2) 2 va 6 sonlari yig‘indisining uchlangani shu sonlar ko‘paytmasidan ikki marta ortiq.

7. Amallar tartibini ko‘rsating va hisoblang:

1) $1,7\cdot 3^2+\frac{2}{3}\cdot 12-15$;

2) $27,7-\left(\frac{1}{2}\right)^2\cdot 100+6,4:0,8$;

3) $48\cdot 0,05-\left(\frac{1}{3}\right)^2\cdot 54+1,7$;

4) $(2,5)^2+15\cdot\frac{3}{5}-0,24:0,6$.

8. Sonli ifodaning qiymatini toping:

1) $\left(\frac{1}{4}+\frac{1}{6}\right)\cdot\left(\frac{2}{5}-\frac{1}{2}\right)$;

2) $\left(\frac{4}{7}-\frac{3}{2}\right)\cdot\left(\frac{1}{13}-\frac{1}{4}\right)$;

3) $4\frac{2}{3}+\frac{1}{4}\cdot\left(1\frac{7}{9}-\frac{1}{9}\right)$;

4) $5\frac{1}{7}-\frac{1}{7}\cdot\left(1\frac{3}{4}+\frac{1}{4}\right)$.

9. Amallarni bajaring:

1) $\frac{0,3\cdot 5^2-15}{3,5+2^2}$;

2) $\frac{4,2:6-3\frac{1}{3}\cdot 0,3}{7,5:0,5}$;

3) $13\frac{1}{3}\cdot(18,1-(3^2+6,1))$;

4) $((7,8:0,3-3^3)+3,1):0,7$.

2-§ *Algebraik ifodalar*

Quyidagi masalani qaraymiz.

1-masala. Biror son o‘ylang, uni 3 ga ko‘paytiring, hosil bo‘lgan natijaga 6 ni qo‘shing, topilgan yig‘indini 3 ga bo‘ling va o‘ylangan sonni ayiring. Qanday son hosil bo‘ladi?

△ Aytaylik, o‘ylangan son 8 bo‘lsin. Barcha amallarni masala shartida ko‘rsatilgan tartibda bajaramiz:

1) $8\cdot 3=24$; 2) $24+6=30$; 3) $30:3=10$; 4) $10-8=2$.

2 soni hosil bo‘ldi.

Bu yechimni qiymati 2 ga teng bo‘lgan $(8\cdot 3+6):3-8$ sonli ifoda shaklida yozish mumkin.

Bordi-yu, agar 5 soni o‘ylangan bo‘lsa, u holda qiymati yana 2 ga teng bo‘lgan $(5\cdot 3+6):3-5$ sonli ifoda hosil qilingan bo‘lar edi.

Biz qanday sonni o‘ylamaylik, natijada 2 soni hosil bo‘laverar ekan-da, degan faraz tug‘iladi. Buni tekshirib ko‘ramiz. O‘ylangan sonni a harfi bilan belgilaymiz va amallarni yana masala shartida ko‘rsatilgan tartibda yozamiz:

$$(a\cdot 3+6):3-a.$$

Arifmetik amallarning bizga ma’lum bo‘lgan xossalaridan foydalanib, bu ifodani soddalashtiramiz:

$$(a\cdot 3+6):3-a=a+2-a=2.\blacktriangle$$

Masalani yechishda istalgan sonni bildiruvchi a harfi, 3 va 6 sonlari, amallar belgilari va qavslardan iborat $(a\cdot 3+6):3-a$ ifoda hosil qilindi. Bu algebraik ifodaga misoldir va u masala shartini matematik tilga o‘tkazish namunasidir.

Yana algebraik ifodalarga misollar keltiramiz:

$$2(m+n),\quad 3a+2ab-7,\quad (a+b)(a-b),\quad \frac{x+y}{a}.$$

Algebraik ifoda sonlar va harflardan tuzilib, amallar belgilari bilan birlashtirilgan ifodadir.

Agar algebraik ifodaga kirgan harflar o‘rniga biror son qo‘yilsa va ko‘rsatilgan amallar bajarilsa, natijada, hosil qilingan son *berilgan algebraik ifodaning son qiymati* deyiladi.

Masalan, $a=2$, $b=3$ bo‘lganda

$$3a+2b-7$$

algebraik ifodaning qiymati 5 ga teng, chunki $3\cdot 2+2\cdot 3-7=5$; shu algebraik ifodaning qiymati $a=1$; $b=0$ bo‘lganda -4 ga teng, chunki

$$3\cdot 1+2\cdot 0-7=-4.$$

a ning istalgan qiymatida

$$(a\cdot 3+6):3-a$$

algebraik ifodaning qiymati 2 ga teng.

2-masala. $\frac{(3a+7)b}{a-b}$ ifodaning qiymatini $a=10$, $b=5$ bo‘lganda toping.

△ $\frac{(3\cdot 10+7)\cdot 5}{10-5}=\frac{37\cdot 5}{5}=37.$ ▲

Mashqlar

10. Algebraik ifodaning qiymatini toping:

1) $3a-2b$, bunda $a=\frac{1}{3}$, $b=1$;

2) $2a+3b$, bunda $a=3$, $b=-2$;

3) $0,25a-4c^2$, bunda $a=4$, $c=3$;

4) $\left(2a^2-\frac{1}{3}b\right)$, bunda $a=2$, $b=9$.

11. Algebraik ifodaning qiymatini toping:

1) $\frac{1}{4}x-\frac{3}{7}y$, bunda $x=8, y=-14$;

2) $\frac{2}{3}x+\frac{4}{5}y$, bunda $x=9,\ y=-10$;

3) $\frac{a-3b}{a+3b}$, bunda $a=4,\ b=-2$;

4) $\frac{a+3c}{2a-c}$, bunda $a=3,\ c=-1$.

12. Neft quvuridan 1 soatda 7 t neft oqadi, m soatda quvurdan necha tonna neft oqib o‘tadi? Bir sutkada-chi?

13. 1) m soatda; 2) p sekundda; 3) m soat l minut va p sekundda necha minut bor?

14. x va y sonlar ayirmasining uchlanganini yozing. Shu ifodaning:

1) $x=-0,37,\ y=-0,42$; 2) $x=-2,98,\ y=-4,48$;

3) $x=-\frac{5}{6},\ y=-\frac{9}{4}$; 4) $x=\frac{2}{15},\ y=-0,7$

bo‘lgandagi son qiymatini toping.

15. x va y sonlar yig‘indisi bilan ular ayirmasining ko‘paytmasini yozing. Hosil bo‘lgan algebraik ifodaning:

1) $x=-\frac{1}{8},\ y=\frac{1}{4}$; 2) $x=-\frac{5}{8},\ y=\frac{3}{4}$;

3) $x=0,15,\ y=-0,75$; 4) $x=1,32,\ y=-1,28$

bo‘lgandagi son qiymatini toping.

Algebraik ifodalarning son qiymatini toping **(16—17):**

16. 1) $\frac{2mn(n+k)}{n-k}$, bunda $m=k=\frac{1}{3},\ n=\frac{1}{2}$;

2) $\frac{(3p+1)\cdot 2p}{p-l}+\frac{1}{3}$, bunda $p=\frac{1}{3},\ l=1$.

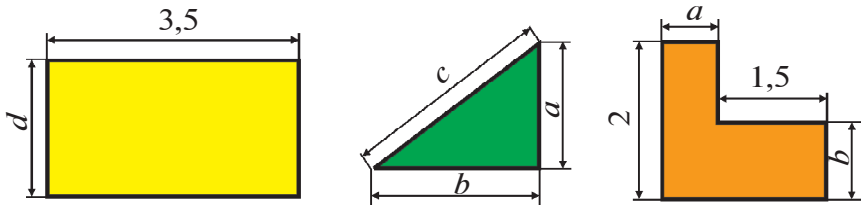
17. 1) $\frac{3(x-y)}{2p+q}$, bunda $x=8,31;\ y=2,29;\ p=2,01;\ q=2$;

2) $\frac{5(bc+m)}{2q+4\frac{1}{4}}$, bunda $b=\frac{2}{3};\ c=6;\ q=\frac{1}{2},\ m=\frac{1}{5}$.

18. Toq son formulasi $n = 2k + 1$ dan foydalanib, $k = 0$, $k = 1$, $k = 7$, $k = 10$ bo‘lganda n ning qiymatini toping.

19. Algebraik ifoda shaklida yozing:
1) kichigi n ga teng bo‘lgan ikkita ketma-ket natural sonning yig‘indisi; 2) kattasi m ga teng bo‘lgan ikkita ketma-ket natural sonning ko‘paytmasi; 3) kichigi $2k$ ga teng bo‘lgan uchta ketma-ket juft natural sonning yig‘indisi; 4) kichigi $2p + 1$ ga teng bo‘lgan uchta ketma-ket toq natural sonning ko‘paytmasi.

20. Shakllarning perimetri va yuzini algebraik ifoda shaklida yozing (1- rasm):



1- rasm.

21. Uyni isitish uchun p tonna ko‘mir g‘amlandi; shu zaxiradan q tonna sarf qilindi. Necha tonna ko‘mir qoldi? 1) $p = 20$, $q = 15$ bo‘lganda hisoblang; 2) q son p sondan katta bo‘lishi mumkinmi? p ga teng bo‘lishi-chi?

22. Kurash musobaqasida har biri 400 so‘mdan n ta chipta va har biri 500 so‘mdan m ta chipta sotildi. Hamma chiptalar uchun qancha pul olingan? $n = 200$, $m = 150$; $n = 100$, $m = 230$ bo‘lganda hisoblang.

23. Bitta albomning bahosi 200 so‘m, bitta daftarning bahosi 40 so‘m, bitta ruchkaning bahosi 60 so‘m. c ta albom, a ta daftar va b ta ruchkaning umumiy (so‘mlardagi) bahosini p harfi bilan belgilab, uni formula shaklida yozing. Agar $c = 9$, $a = 21$, $b = 4$ bo‘lsa, bu formula bo‘yicha p ni hisoblang.

24. Issiqlik uzatish stansiyasi uchun mo‘ljallangan gaz quvuri orqali har minutda 26 m^3 gaz o‘tadi. 5 sutkada; m sutkada quvurdan necha kub metr gaz o‘tadi?

25. Geologlar o‘z yo‘nalishi bo‘yicha harakat qilib, otda soatiga c kilometr tezlik bilan 3 soat-u 10 minut yurishdi; oqimining tezligi soatiga a kilometr bo‘lgan daryoda oqim bo‘yicha 1 soat-u 40 minut solda suzishdi va soatiga b kilometr tezlik bilan 2 soat-u 30 minut piyoda yurishdi. Yo‘nalishning (km lardagi) uzunligini s harfi bilan belgilab, geologlar bosib o‘tgan yo‘l formulasini yozing. Agar $a = 3{,}3$ km/soat, $b = 5{,}7$ km/soat, $c = 10{,}5$ km/soat bo‘lsa, yo‘nalishning uzunligini hisoblang.

3-§ *Algebraik tengliklar, formulalar*

Ko‘pgina amaliy masalalarni yechishda sonlarni belgilash uchun harflardan foydalanish qulaydir.

Masalan, agar a va b to‘g‘ri to‘rtburchak tomonlarining uzunliklari bo‘lsa, u holda $a \cdot b$ — uning yuzi, $2 \cdot (a + b)$ — uning perimetri. Bu yerda a va b harflari bilan musbat sonlar — to‘g‘ri to‘rtburchakning tomonlarining uzunliklari belgilangan. Agar to‘g‘ri to‘rtburchak yuzini S harfi bilan, perimetrini esa P bilan belgilasak, u holda quyidagi formulalarni hosil qilamiz:

$$S = a \cdot b, \quad P = 2 \cdot (a + b).$$

Agar tomonlar uzunliklari santimetrlarda o‘lchangan bo‘lsa, u holda S yuz kvadrat santimetrlarda, P perimetr esa santimetrlarda ifodalanadi.

Yozuvni qisqartirish uchun ko‘paytirish belgisi — „nuqta“ ko‘pincha tushirib qoldiriladi. Masalan, $S = ab$, $P = 2(a + b)$ deb yoziladi.

Harflar bilan, shuningdek, tenglamalardagi noma’lum sonlar ham belgilanadi. Masalan: $x + 12{,}3 = 95{,}1$ tenglamadagi noma’lum son x harfi bilan belgilangan, $2y + 3 = 7$ tenglamadagi noma’lum son esa y harfi bilan belgilangan.

Harflar bilan arifmetik amallar qonunlari va xossalarini yozish ham qulaydir. Masalan:

$$a - (b + c) = (a - b) - c = a - b - c, \quad (1)$$
$$(a + b) \cdot c = a \cdot c + b \cdot c, \quad (2)$$
$$(a + b) : c = a : c + b : c. \quad (3)$$

XVI asrning taniqli matematigi Fransua Viyet (1540—1603) algebraga harfiy belgini kiritishning asoschisi hisoblanadi.

Algebrada birgina harfning o‘zi har xil sonli qiymatlar qabul qilishi mumkin. Jumladan, (1) va (2) tengliklarda a, b, c — ixtiyoriy sonlar; (3) tenglikda esa a, b — istalgan sonlar, lekin $c \neq 0$, chunki nolga bo‘lish mumkin emas.

Harflar yordamida juft va toq natural sonlar formulasini yozish mumkin.

Agar a juft son bo‘lsa, u holda bu son 2 ga bo‘linadi va uni bunday yozish mumkin:

$$a = 2n,$$

bu yerda n — natural son.

Agar b toq son bo‘lsa, u holda uni 2 ga bo‘lgandagi qoldiq 1 ga teng, binobarin, b sonni bunday yozish mumkin:

$$b = 2n + 1,$$

bu yerda n — natural son yoki nol.

Ba’zan, toq natural sonlar formulasini quyidagicha ham yozishadi:

$$b = 2k - 1,$$

bu yerda k — natural son.

Formulalar boshqa fanlarda ham bor. H_2O — suvning, Og_{3+3} Ch_{3+3} $U_{(3)}$ lola gulining formulasi ekanini kimyo, botanika darslarida o‘rgangansiz.

Harflardan foydalanish bir xil toifadagi ko‘pgina masalalarni yechish yo‘lini yozishga imkon beradi. Shunga doir masalalar qaraylik.

1- masala. Fermerning bog‘ maydoni to‘g‘ri to‘rtburchak shaklida bo‘lib, uning bo‘yi a kilometrga, eni esa b kilometrga

teng. Yangi yer o‘zlashtirilgandan keyin maydonning yuzi 0,88 km^2 ga ortdi. Bog‘ maydonining yuzi qancha bo‘ldi? Hisoblashlarni: 1) $a = 2{,}2$ va $b = 0{,}8$; 2) $a = 1{,}4$ va $b = 4{,}3$ uchun bajaring.

△ Dastlab bog‘ning yuzi $a \cdot b$ km^2 ga teng edi, yangi yer ochilgandan keyin u $(ab + 0{,}88)$ km^2 ga teng bo‘ldi.

1) $a = 2{,}2$ va $b = 0{,}8$ bo‘lganda, $2{,}2 \cdot 0{,}8 + 0{,}88 = 2{,}64$.

2) $a = 1{,}4$ va $b = 4{,}3$ bo‘lganda, $1{,}4 \cdot 4{,}3 + 0{,}88 = 6{,}9$. ▲

2- masala. Sayyoh qishloqdan chiqib, shahar tomon jo‘nadi. U a kilometr piyoda yurganidan keyin avtobusga o‘tirdi va avtobusda t soatda shaharga yetib keldi. Agar avtobus 60 km/soat tezlik bilan harakat qilgan bo‘lsa: 1) $a = 5$ va $t = 0{,}5$ bo‘lganda qishloq bilan shahar orasidagi s masofani hisoblang; 2) $s = 70$, $a = 10$ bo‘lganda t ni toping.

△ Sayyoh avtobusda t soatda $60t$ kilometr yo‘l bosgan. Shuning uchun qishloq bilan shahar orasidagi masofa

$$s = a + 60t$$

formula bilan ifodalanadi.

1) $a = 5$ va $t = 0{,}5$ bo‘lganda, $s = 5 + 60 \cdot 0{,}5 = 35$ km bo‘ladi;

2) $s = a + 60t$ formuladan t ni topamiz: $t = \frac{s - a}{60}$. Bu yerdan $s = 70$, $a = 10$ bo‘lganda, $t = \frac{70 - 10}{60} = 1$ soat. ▲

Mashqlar

26. Jumlalarni matematik tilda yozing:

1) m va n sonlarning yig‘indisini;
2) a va b sonlarning ayirmasini;
3) a va b sonlar ayirmasining ikkilanganini;
4) m va n sonlar ko‘paytmasining ikkilanganini;
5) n va m sonlar yig‘indisining ular ayirmasiga bo‘linmasini;
6) a va b sonlar yig‘indisining ular ayirmasiga ko‘paytmasini.

27. Quyidagi ifodalarda harflar qanday sonlarni ifodalashi mumkin:

1) tanaffus n minut davom etadi;
2) sinfimizda y nafar o‘quvchi bor;

3) 7- sinfda x ta o‘quv fani o‘qitiladi;
4) bir oyda k kun bor?

28. Yerning sun’iy yo‘ldoshi 9 km/s tezlik bilan harakat qiladi. Ushbu jadvalni to‘ldiring:

Bosib o‘tilgan masofa, km	45 000	1 350 000
Harakat vaqti, s		

29. „Spark“ avtomobili 100 km yo‘lga a litr yonilg‘i sarf qiladi. Ushbu jadvalni to‘ldiring:

Bosib o‘tilgan masofa, km	300	800	1000			s
Yonilg‘i sarfi, l				$5a$	$4a$	

30. Birinchi qopda m kilogramm, ikkinchi qopda esa birinchi qopdagidan n kilogramm kam un bor. Ikkinchi qopda necha kilogramm un bor? Masalani 1) $m = 50$ va $n = 12$; 2) $m = 45$ va $n = 15$ hollar uchun yeching.

31. Piyoda 1 soatda 5 km yo‘l bosadi. U: 1) 3 soatda necha kilometr yo‘l bosadi? 2) k soatda-chi?

32. Do‘konga har birida 50 kg dan un bo‘lgan a ta qop keltirildi. Do‘konga necha kilogramm un keltirilgan?

33. Bog‘bonlar 1 kunda 15 gektar bog‘ga ishlov berishdi. Ular a kunda necha gektar bog‘ga ishlov berishadi?

34. Har biri x so‘mdan 6 ta daftar va har biri y so‘mdan 3 o‘ram qog‘oz sotib olindi. Hamma xarid qancha turadi?

35. Yuk mashinasi do‘konga ombordan har biri a kilogrammdan 15 yashik olxo‘ri va har biri b kilogrammdan 20 yashik olma keltirdi. Do‘konga necha kilogramm meva keltirilgan?

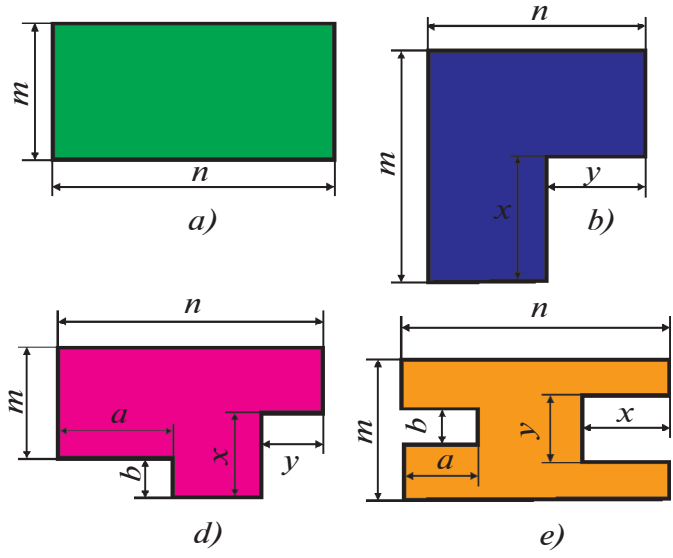
36. Mashinaga har biri m kilogrammdan k qop bug‘doy va har biri n kilogrammdan c qop arpa yuklandi. Mashinaga necha kilogramm don yuklangan?

37. To‘g‘ri to‘rtburchak shaklidagi tajriba maydonining bo‘yi a metrga teng, eni esa bo‘yidan b metr qisqa. Shu maydonning yuzi S ning formulasini yozing.

38. Kinoteatrda har biri n ta o‘rindiqqa ega bo‘lgan m ta qator va yana k ta qo‘shimcha o‘rindiq bor. Kinoteatrda hammasi bo‘lib nechta o‘rindiq bor? Masalani yechish formulasini tuzing va $m = 30$, $n = 25$, $k = 60$ bo‘lganda hisoblashlarni bajaring.

39. Dars jadvalida 5 ta dars, ikkita 15 minutlik va ikkita 10 minutlik tanaffus bo‘lgan kuni o‘quvchi maktabda necha soat bo‘ladi? (1 dars — 45 minut.)

40. O‘lchamlari 2- rasmda ko‘rsatilgan shakllarning perimetrlarini va yuzlarini hisoblash uchun formulalar yozing:



2- rasm.

41. To‘g‘ri to‘rtburchakning bo‘yi kvadratning tomonidan 8 m uzun, eni esa shu kvadrat tomonidan 4 m qisqa. Kvadrat tomonini biror harf bilan belgilab, to‘g‘ri to‘rtburchak uchun: 1) tomonlarning uzunligini; 2) perimetrini; 3) yuzini yozing.

42. Avtobus t soatda s kilometr yo‘l bosadi. Avtomobil xuddi shu yo‘lni avtobusdan 1 soat oldin bosib o‘tishi uchun qanday tezlikka ega bo‘lishi kerak?

43. $x = 2a + 3b$ (km) formula avtobusning harakati haqidagi masala yechilishini bildiradi. Masala shartini tuzing.

44. Maktab tajriba maydoni a kvadrat metr yuzga ega. Bog‘ yuzi 1500 m^2 bo‘lgan joyni egallagan, qolgan maydon 20 ta bir xil maydonchaga bo‘lingan. Shu maydonchalarning har biri qanday yuzga ega?

45. Bankka 50 000 so‘m pul qo‘yildi. Bir yildan so‘ng jamg‘arma $p\,\%$ ko‘paydi. Bir yildan keyin jamg‘armaning miqdori necha so‘mga yetdi?

46. Asosi a detsimetr, perimetri esa 42 dm bo‘lgan to‘g‘ri to‘rtburchakning yuzini hisoblash uchun ifoda tuzing. a ning ushbu jadvalda keltirilgan qiymatlari uchun to‘g‘ri to‘rtburchak yuzi S ning qiymatini (dm^2 larda) hisoblang:

a	5	6	7,5	10	12	12,5	15
S							

№ 1 *Faqat 4 ta 9 va arifmetik amal belgilari yordamida qiymati 100 ga teng bo‘lgan sonli ifoda tuzing.*

47. Velosipedchi soatiga v kilometr tezlik bilan harakat qilmoqda. U jo‘nash joyidan s kilometr uzoqlikda bo‘lgan qishloqqa borishi kerak. Agar u 3 km yo‘lni bosib o‘tgan bo‘lsa, unga qishloqqa yetib borishi uchun yana qancha vaqt talab qilinadi? Agar u 3 km yurgan va $s = 36$ km, $v = 12$ km/soat bo‘lsa, qishloqqa 2,5 soatda yetib bora oladimi?

48. Bir avtomobil 100 km yo‘lga o‘rtacha 5 l, ikkinchi avtomobil esa 100 km yo‘lga o‘rtacha 10 l benzin sarflaydi. Har bir avtomobil bakida a litr benzin bo‘lsa, ular qanday masofaga bora oladi? Agar $a = 20$ l va avtomobillar Toshkentdan bir vaqtda Samarqandga qarab yo‘lga chiqishgan bo‘lsa, qaysi mashina Samarqandga yetib kela oladi? (Toshkent va Samarqand shaharlari orasidagi masofa 300 km.)

4-§ Arifmetik amallarning xossalari

Algebrani puxta o‘rganish uchun arifmetik amallarning xossalarini yaxshi bilish lozim. Eslatib o‘taylik, arifmetik amallar deb qo‘shish, ayirish, ko‘paytirish va bo‘lish amallariga aytiladi. Sonlar ustidagi bu amallarning xossalarini qisqacha formulalar ko‘rinishida yozamiz. Amallarning asosiy xossalari, odatda, *qonunlar* deb ataladi. Qonunlardan foydalanib, amallarning boshqa xossalarini ham asoslash mumkin.

1. Qo‘shish va ko‘paytirish.

Qo‘shish va ko‘paytirishning asosiy qonunlarini sanab o‘tamiz.

1. *O‘rin almashtirish qonuni:*

$$a+b=b+a,\ \ ab=ba.$$

2. *Guruhlash qonuni:*

$$(a+b)+c=a+(b+c),\ \ (ab)c=a(bc).$$

3. *Taqsimot qonuni:*

$$a(b+c)=ab+ac.$$

Bu tengliklarda a, b, c — ixtiyoriy sonlar. Masalan:

$$1,2+3,5=3,5+1,2;\quad \frac{3}{4}\cdot\left(-\frac{2}{7}\right)=\left(-\frac{2}{7}\right)\cdot\frac{3}{4};$$

$$(-8)\cdot(125+7)=(-8)\cdot 125+(-8)\cdot 7.$$

Qo‘shish va ko‘paytirish qonunlari yordamida amallarning boshqa xossalarini ham hosil qilish mumkin. Masalan:

$$a+b+c+d=a+(b+c+d),\ (abc)d=(ab)(cd),$$
$$(a+b+c)d=ad+bd+cd.$$

1- masala. Hisoblang: $75 + 37 + 25 + 13$.

△ Hisoblashlarni ko‘rsatilgan tartibda olib borish mumkin: 75 ga 37 ni qo‘shib, natijaga 25 ni qo‘shish va oxirgi natijaga 13 ni qo‘shish. Lekin qo‘shishning xossalaridan foydalanib, hisoblashlarni soddalashtirish mumkin:

$$75+37+25+13=(75+25)+(37+13)=100+50=150.$$ ▲

Bu misol shuni ko‘rsatadiki, amallarning xossalaridan foydalanib, hisoblashlarni eng sodda (oqilona) usulda bajarish mumkin.

Amallarning xossalari algebraik ifodalarni soddalashtirish maqsadida bajariladigan almashtirishlarda ham qo‘llaniladi.

2- masala. Ifodani soddalashtiring:

$$3(2a+4b)+5(7a+b).$$

△ $3(2a+4b)+5(7a+b)=3\cdot 2a+3\cdot 4b+5\cdot 7a+5\cdot b=6a+12b+35a+5b=$

$$=(6a+35a)+(12b+5b)=(6+35)a+(12+5)b=41a+17b.$$ ▲

Bu masalani yechish jarayonida quyidagi ifoda hosil bo‘ldi:

$$6a+12b+35a+5b.$$

Bu ifodada $6a$ va $35a$ qo‘shiluvchilar o‘xshashdir, chunki ular bir-biridan faqat koeffitsiyentlari bilangina farq qiladi. $12b$ va $5b$ qo‘shiluvchilar ham o‘xshash. Shu sababli $6a + 12b + 35a + 5b$ ifoda o‘rniga $41a + 17b$ ifodani yozish, ya’ni o‘xshash hadlarni ixchamlash mumkin bo‘ladi.

Oraliq hisoblashlarni og‘zaki bajarib, almashtirishlar yozuvini qisqartirish mumkin. Masalan,

$$6(3x+4)+2(x+1)=18x+24+2x+2=20x+26.$$

2. Ayirish.

3- masala. Toshkent va Samarqand shaharlari orasida Jizzax shahri joylashgan. Toshkentdan Samarqandgacha bo‘lgan masofa 300 km, Toshkentdan Jizzaxgacha bo‘lgan masofa esa 180 km. Jizzaxdan Samarqandgacha bo‘lgan masofani toping.

△ Jizzaxdan Samarqandgacha bo‘lgan masofa x kilometr bo‘lsin. U holda

$180+x=300$, bu yerdan $x=300-180=120$.

Javob: 120 km.▲

$180 + x = 300$ tenglikdan x qo‘shish amaliga teskari deb ataluvchi ayirish amali yordamida topiladi.

a sondan b sonni ayirish uchun a songa b songa qarama-qarshi bo‘lgan sonni qo‘shish kifoya:

$$a - b = a + (-b).$$

Shu sababli ayirish amalining xossalarini qo‘shish amalining xossalari orqali asoslash mumkin. Masalan:

$$251 + (49 - 13) = 251 + 49 - 13 = 287, \qquad a + (b - c) = a + b - c,$$
$$123 - (23 + 39) = 123 - 23 - 39 = 61, \qquad a - (b + c) = a - b - c,$$
$$123 - (83 - 77) = 123 - 83 + 77 = 117, \qquad a - (b - c) = a - b + c.$$

4- masala. Ifodaning qiymatini hisoblang:

$$4(3x - 5y) + 6(x - y),$$

bunda $x = \frac{1}{2}$, $y = \frac{1}{13}$.

△ Avval berilgan ifodani soddalashtiramiz:

$$4(3x - 5y) + 6(x - y) = 12x - 20y + 6x - 6y = 18x - 26y.$$

Hosil bo‘lgan ifodaning $x = \frac{1}{2}$, $y = \frac{1}{13}$ dagi qiymatini hisoblaymiz:

$$18 \cdot \frac{1}{2} - 26 \cdot \frac{1}{13} = 9 - 2 = 7.\ ▲$$

Amallarning xossalaridan foydalanish algebraik ifodani avval soddalashtirib, so‘ngra uning qiymatini oson yo‘l bilan hisoblash imkonini beradi.

3. Bo‘lish.

5- masala. To‘g‘ri to‘rtburchakning yuzi 380 sm^2, tomonlaridan biri 95 sm. To‘g‘ri to‘rtburchakning ikkinchi tomoni uzunligini toping.

△ $S = ab$ formuladan $b = \frac{S}{a}$ ni topamiz. $S = 380\ sm^2$, $a = 95$ sm bo‘lgani uchun

$$b = \frac{380\ sm^2}{95\ sm} = 4\ sm.$$

Javob: 4 sm.▲

$ab = S$ tenglikdan b ko‘paytirish amaliga teskari deb ataluvchi bo‘lish amali yordamida topiladi.

a sonni b songa bo‘lish uchun a sonni b soniga teskari bo‘lgan songa ko‘paytirish kerak:

$$\frac{a}{b} = a : b = a \cdot \frac{1}{b}.$$

Shu sababli bo‘lish amalining xossalarini ko‘paytirishning xossalaridan keltirib chiqarish mumkin.

6- masala. Tenglikni isbotlang:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c},$$

bu yerda $c \neq 0$.

○ Bo‘lishni ko‘paytirish bilan almashtirib, quyidagini hosil qilamiz:

$$\frac{a+b}{c} = (a+b) \cdot \frac{1}{c}.$$

Taqsimot qonunini qo‘llab,

$$(a+b) \cdot \frac{1}{c} = a \cdot \frac{1}{c} + b \cdot \frac{1}{c}$$

ni topamiz. Ko‘paytirishni bo‘lish bilan almashtirib,

$$a \cdot \frac{1}{c} + b \cdot \frac{1}{c} = \frac{a}{c} + \frac{b}{c}$$

ni hosil qilamiz. ●

Mashqlar

49. Arifmetik amallar qonunlari va xossalarini qo‘llab, sonli ifodaning qiymatini toping:

1) $29 \cdot 0,45 + 0,45 \cdot 11$;

2) $(51,8 + 44,3 + 48,2 - 24,3) \cdot \frac{1}{3}$;

3) $4,07 - 5,49 + 8,93 - 1,51$;

4) $-11,401 - 23,17 + 4,401 - 10,83$.

50. O‘xshash hadlarni ixchamlang:

1) $4a+2b+a-b$;
2) $x-2y-3x+5y$;
3) $0,1c-0,3+d-c-2,1d$;
4) $8,7-2m+n-\frac{1}{3}m+\frac{2}{3}n$.

51. O‘xshash hadlarni ixchamlang:

1) $2,3a-0,7a+3,6a-1$;
2) $0,48b+3+0,52b-3,7b$;
3) $\frac{1}{3}x+\frac{1}{2}x-\frac{1}{6}a-\frac{5}{6}a+2$;
4) $\frac{5}{6}y-\frac{1}{3}b-\frac{1}{6}y+\frac{2}{3}b-3$;
5) $2,1m+n-3,2n+2m+1,1m-n$;
6) $5,7p-2,7q+0,3p+0,8q+1,9q-p$.

52. Ifodani soddalashtiring:

1) $3(2x+1)+5(1+3x)$;
2) $4(2+x)-3(1+x)$;
3) $10(n+m)-4(2m+7n)$;
4) $11(5c+d)+3(d+c)$.

53. Ifodani soddalashtiring va son qiymatini toping:

1) $5(3x-7)+2(1-x)$, bunda $x=\frac{1}{26}$;
2) $7(10-x)+3(2x-1)$, bunda $x=-0,048$;
3) $\frac{1}{3}(6x-3)+\frac{2}{5}(5x-15)$, bunda $x=3,01$;
4) $0,01(2,2x-0,1)+0,1(x-100)$, bunda $x=-10$.

54. Arifmetik amallarning xossalaridan foydalanib hisoblang:

1) $\frac{1}{7}(0,14+2,1-3,5)$;
2) $\frac{1}{12}(4,8-0,24-1,2)$;
3) $(18\frac{6}{7}+21\frac{3}{4}):3$;
4) $(15\frac{5}{7}+20\frac{15}{16})\cdot\frac{1}{5}$.

5-§ *Qavslarni ochish qoidalari*

1. Algebraik yig‘indi.

1-masala. Yigirma qavatli binoda lift ishlamoqda. U 8-qavatdan 6 qavat pastga tushdi, so‘ngra 12 qavat yuqoriga ko‘tarildi, 4 qavat pastga tushdi, 7 qavat yuqoriga ko‘tarildi, 13 qavat pastga tushdi. Lift qaysi qavatda turibdi?

△ Liftning qaysi qavatda turganligini topish uchun $8-6+12-4+7-13$ ifodaning qiymatini hisoblash kerak. Bu qiymat 4 ga teng. Demak, lift 4- qavatda turibdi. ▲

Siz 6- sinf matematika kursidan

$$8-6+12-4+7-13$$

ifoda algebraik yig‘indi deb atalishini bilasiz, chunki uni yig‘indi shaklida bunday yozish mumkin:

$$8+(-6)+12+(-4)+7+(-13).$$

Algebraik yig‘indilarga oid yana misollar keltiramiz:

$$3-(-7)+(-2),\quad a-b+c-d,\quad a+(-b)-(-c).$$

$(-c)$ sonni ayirish $(-c)$ songa qarama-qarshi sonni, ya’ni c sonni qo‘shishni bildirishini eslatib o‘tamiz. Shuning uchun oxirgi algebraik yig‘indini bunday yozish mumkin:

$$a+(-b)+c.$$

Algebraik yig‘indi — bu „+“ va „−“ ishoralari bilan birlashtirilgan bir nechta algebraik ifodalardan tuzilgan yozuvdir.

Odatda, $3-(-7)+(-2)$, $a+(-b)-(-c)$ ko‘rinishidagi algebraik yig‘indilar qisqacha bunday yoziladi:

$$3-(-7)+(-2)=3+7-2;\quad a+(-b)-(-c)=a-b+c.$$

$3+7-2$ algebraik yig‘indida qo‘shiluvchilar 3, 7 va -2 sonlari bo‘ladi, chunki $3+7-2=3+7+(-2)$; $a-b+c$ algebraik yig‘indida qo‘shiluvchilar a, $-b$, c sonlar bo‘ladi, chunki $a-b+c=a+(-b)+c$.

2. Qavslarni ochish va qavs ichiga olish.

$a+(b+c)$ ifodani qaraymiz: qo‘shishning guruhlash qonunini qo‘llab, uni bunday yozish mumkin:

$$a+(b+c)=a+b+c.$$

Bu tenglikda c ni $-d$ bilan almashtiramiz:

$$a+(b-d)=a+b-d.$$

Qavs oldida „+“ ishorasi turgan ifodalarda almashtirishlar bajarish shu tengliklarga asoslangan. Bu tengliklar qavslarni ochishning quyidagi birinchi qoidasiga olib keladi:

Agar algebraik ifodaga qavs ichiga olingan algebraik yig‘indi qo‘shiladigan bo‘lsa, u holda shu algebraik yig‘indidagi har bir qo‘shiluvchining ishorasini saqlagan holda qavslarni tushirib qoldirish mumkin.

Masalan:

1) $14+(7-13+2)=14+7-13+2$;
2) $a+(b+c-d)=a+b+c-d$;
3) $(a-b)+c=a-b+c$.

Qavs oldida „–“ ishorasi turgan ifodalarda almashtirishlar bajarish ayirish amalining quyidagi xossalariga asoslangan:

$$-(-a)=a,\quad -(a+b)=-a-b,$$
$$a-(b+c)=a-b-c,$$
$$a-(b-c)=a-b+c.$$

Bu tengliklardan *qavslarni ochishning* quyidagi *ikkinchi qoidasi* kelib chiqadi:

Agar algebraik ifodadan qavs ichiga olingan algebraik yig‘indi ayirilsa, u holda shu algebraik yig‘indidagi har bir qo‘shiluvchining ishorasini qarama-qarshisiga o‘zgartirib, qavslarni tushirib qoldirish mumkin.

Masalan:

1) $14-(7-13+2)=14-7+13-2$;
2) $a-(b+c-d)=a-b-c+d$;
3) $-(a-b)+c=-a+b+c$.

2- masala. Qavslarni ochib soddalashtiring:

$$3x+(5-(8x+3)).$$

△ $3x+(5-(8x+3))=3x+5-(8x+3)=3x+5-8x-3=2-5x$. ▲

Ba'zan bir necha qo'shiluvchini qavs ichiga olish foydali bo'ladi.

Masalan:

1) $108-137+37=108-(137-37)=108-100=8$;

2) $\boxed{a+b-c+d=a+(b-c+d).}$

Bu yerda qavs oldiga „+" belgisi qo'yilgan, shuning uchun qavs ichidagi barcha qo'shiluvchilarning ishoralari saqlanib qoladi.

3) $\boxed{a-b-c+d=a-(b+c-d).}$

Bu yerda qavs oldiga „–" belgisi qo'yilgan, shuning uchun qavs ichiga olingan barcha qo'shiluvchilarning ishoralari qarama-qarshisiga o'zgartirildi.

Mashqlar

55. Algebraik yig'indini qavslarsiz yozing:

1) $(+4)+(-3)-(+7)$;
2) $(-4)+(-9)-(-11)$;
3) $(-a)+(-7b)+\frac{1}{3}c$;
4) $2a+(-3b)-4c$.

56. Algebraik yig'indining qo'shiluvchilarini ayting:

1) $15-c$; 2) $m-7$; 3) $-a+47$; 4) $-13-b$.

57. Algebraik yig'indi shaklida yozing:

1) $a-b+c$; 2) $2+b-c$; 3) $a-2-b$; 4) $3+a-b-c$.

Qavslarni oching **(58—59)**:

58. 1) $a+(2b-3c)$;
2) $a-(2b-3c)$;
3) $a-(2b+3c)$;
4) $-(a-2b+3c)$.

59. 1) $a+(b-(c-d))$;
2) $a-(b-(c-d))$;
3) $a-((b-c)-d)$;
4) $a-(b+(c-(d-k)))$.

60. Qavslarni oching va soddalashtiring:

1) $3a-(a+2b)$;
2) $5x-(2y-3x)$;
3) $3m-(5m-(2m-1))$;
4) $4a+(2a-(3a+3))$.

61. m yoki $(-m)$ sonlaridan boshlab, barcha qo‘shiluvchilarni qavs oldiga „+“ ishorasini qo‘ygan holda qavs ichiga oling:

1) $a+2b+m-c$;
2) $a-2b+m+c$;
3) $a-m+3c+4d$;
4) $a-m+3b^2-2a^3$.

62. m yoki $(-m)$ sonlaridan boshlab, barcha qo‘shiluvchilarni qavs oldiga „–“ ishorasini qo‘ygan holda qavs ichiga oling:

1) $2a+3b+m-c$;
2) $2a+b+m+3c$;
3) $c-m-2a+3b^2$;
4) $a-m+3b^2-2a^3$.

63. 1) $a+b-1$ ifodani biri a ga teng bo‘lgan ikkita qo‘shiluvchining yig‘indisi shaklida yozing;
2) $a-b+1$ ifodani kamayuvchisi a bo‘lgan ayirma shaklida yozing;
3) $2a-b+4$ ifodani kamayuvchisi $2a$ bo‘lgan ayirma shaklida yozing;
4) $a-2b+8$ ifodani biri 8 ga teng bo‘lgan ikkita qo‘shiluvchining yig‘indisi shaklida yozing.

64. Tengliklarning chap qismlari bir xil. Nega o‘ng qismlari har xil? Qanday shartlarda tenglik o‘rinli bo‘ladi?
1) $2400+750:15-40\cdot 3=2330$;
2) $2400+750:15-40\cdot 3=90$;
3) $2400+750:15-40\cdot 3=2430$;
4) $2400+750:15-40\cdot 3=2310$;
5) $2400+750:15-40\cdot 3=7210$;
6) $2400+750:15-40\cdot 3=2407$;
7) $2400+750:15-40\cdot 3=510$.

65. Ko‘p nuqtalar o‘rniga „+“ va „–“ ishoralarini shunday qo‘yingki, natijada to‘g‘ri tenglik hosil bo‘lsin:

1) $a-(b+c)=a+(\ldots b\ \ldots c)$;
2) $c-(a-b)=c+(\ldots a\ \ldots b)$;
3) $m-(n-a)=m+(\ldots n\ \ldots a)$;
4) $n-(d-l)=n+(\ldots d\ \ldots l)$.

O‘zingizni tekshirib ko‘ring!

1. Hisoblang:

1) $(17{,}2\cdot 4{,}01+4{,}01\cdot 32{,}8):1\frac{2}{3}$;

2) $\frac{1}{2}-\left(\frac{1}{2}\right)^2\cdot 2\left(\frac{2}{3}\right)-25\cdot 0{,}03\cdot 4.$

2. Ifodani soddalashtiring va $x=-\frac{2}{9}$, $y=0{,}25$ bo‘lganda uning son qiymatini toping:
$3(2y-x)-2(y-3x).$

3. Bolalar oromgohi uchun 10 ta shaxmat va 15 ta koptok sotib olishdi. Bitta shaxmat a so‘m, bitta koptok b so‘m turadi. Jami xarid uchun qancha pul to‘langan?

66. Soddalashtiring:

1) $(5a-2b)-(3b-5a)$;
2) $(6a-b)-(2a+3b)$;
3) $7x+3y-(-3x+3y)$;
4) $8x-(3x-2y)-5y.$

67. Tenglamani yeching:

1) $(2x+1)+3x=16$;
2) $(x-4)+(x+6)=4$;
3) $(x-5)-(5-3x)=2$;
4) $23-(x+5)=13.$

68. Ifodani avval soddalashtirib, keyin uning son qiymatini toping:

1) $(2c+5d)-(c+4d)$, bunda $c=0{,}4$, $d=0{,}6$;
2) $(3a-4b)-(2a-3b)$, bunda $a=0{,}12$, $b=1{,}28$;
3) $(7x+8y)-(5x-2y)$, bunda $x=-\frac{3}{4}$, $y=0{,}025$;
4) $(5c-6b)-(3c-5b)$, bunda $c=-0{,}25$, $b=2\frac{1}{2}$.

Algebraik ifodaning son qiymatini hisoblang **(69—75)**:

69. 1) $a+bc$, bunda $a=-1,\ b=3,\ c=0$;
2) $a-bc$, bunda $a=2,\ b=-1,\ c=-3$;
3) $(a+b)c$, bunda $a=1,\ b=-3,\ c=2$;
4) $(a-b)c$, bunda $a=3,\ b=1{,}2,\ c=5$;
5) $(a-b)+(c-d)$, bunda $a=4,\ b=2,\ c=3,\ d=-1$;
6) $(a-b)-(c-d)$, bunda $a=0,\ b=-4,\ c=-2,\ d=3$;
7) $a-(b-c)$, bunda $a=0{,}5,\ b=\frac{1}{2},\ c=-1{,}2$;
8) $a-(b-c)-d$, bunda $a=5{,}2,\ b=1{,}3,\ c=2{,}8,\ d=2{,}8$.

70. 1) $5(x-y)^2$; 2) $3(x+y)^2$; 3) $(5x-y)^2$; 4) $(3x+y)^2$, bunda $x=2{,}5,\ y=4{,}5$.

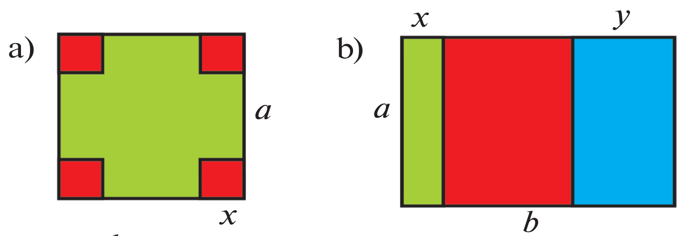
71. 1) $2((a-b)^2+1)$; 3) $((a-b)a-8):2$;
2) $4(3-(a-b)^2)$; 4) $(5a-(a+b)):3$, bunda $a=5,\ b=-1$.

72. 1) $3(a+b)-2ab$; 3) $3(a-b)+2ab$;
2) $3a+b-2ab$; 4) $3a-b+2ab$, bunda $a=1{,}2,\ b=1{,}8$.

73. 1) $\frac{1}{2}b^3-3c^2$, bunda $b=-2,\ c=-\frac{1}{3}$;
2) $-0{,}75a^2+1\frac{2}{3}b^2$, bunda $a=-2,\ b=3$;
3) $(a^2-26)^2$, bunda $a=-5$; 4) $(a^3+26)^3$, bunda $a=-3$.

74. Ifodalarning geometrik ma'nosini oching.
1) $a\cdot b$, bunda a va b — to'g'ri to'rtburchakning tomonlari;
2) a^2, bunda a — kvadratning tomoni uzunligi;
3) $2(a+b)$, bunda a va b — to'g'ri to'rtburchakning tomonlari uzunligi;
4) $4a$, bunda a — kvadratning tomoni.

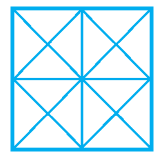
75. 1) a^2-4x^2, bunda a — katta kvadratning tomoni, x — har bir kichik kvadratchaning tomoni uzunligi (3-a rasm);



3- rasm.

2) $\frac{ab}{ax+ay}$, bunda a va b katta to‘g‘ri to‘rtburchakning, x va y esa kichik to‘g‘ri to‘rtburchaklarning tomonlari (3-*b* rasm).

№ 2 *4- rasmda nechta uchburchak, kvadrat va to‘g‘ri to‘rtburchak bor?*

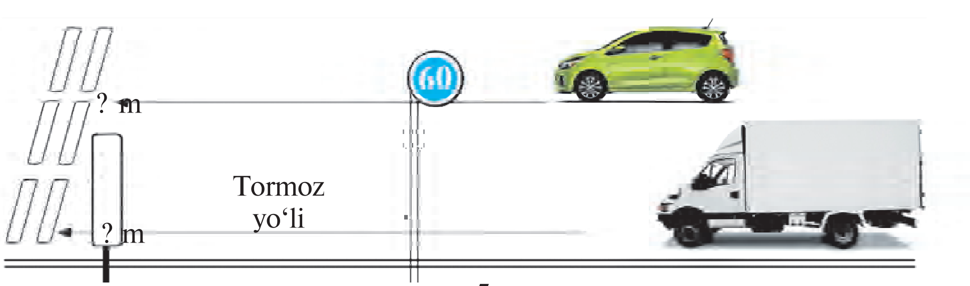


4- rasm.

76. Bir gektar ko‘kalamzor bir yil davomida havoni 70 t changdan tozalashga qodir. 10 ga; 100 ga; *m* gektar ko‘kalamzor bir yilda havoni necha tonna changdan tozalaydi? Umumiy maydoni 16 000 ga bo‘lgan ko‘kalamzor havoni necha tonna changdan tozalaydi?

77. Avtomobilning harakat tezligi ikki marta ortishi bilan uning tormozlanish yo‘lining to‘rt marta ortishi ma’lum. Harakat tezligi 30 km/soat bo‘lganda, tormozlanish yo‘lining uzunligi jadvalda berilgan. Tezlik 60 km/soat bo‘lganda, tormozlanish yo‘lining uzunligi qancha bo‘ladi (5-rasm).

Yuk mashinasi uchun		Yengil mashina uchun	
v (km/soat)	s (m)	v (km/soat)	s (m)
30	9,5	30	7,25



5- rasm.

78. *(Abu Rayhon Beruniy masalasi.)* Agar 10 dirham pul ikki oyda 5 dirham foyda keltirgan bo‘lsa, 8 dirham puldan uch oyda qancha foyda olish mumkin?

I bobga doir sinov mashqlari — testlar

1. $a=5{,}1$, $b=4{,}7$ bo‘lsa, $P=2(a+b)$ ifodaning son qiymatini toping.

A) 196; B) 19,6; C) 1,96; D) 18,16.

2. To‘g‘ri to‘rtburchakning yuzi S ga, asosi a ga teng. Uning perimetrini topish uchun ifoda tuzing.

A) $\frac{S}{2a}+a$; B) $\frac{S}{a}+2a$; C) $2\left(\frac{S}{a}+a\right)$; D) $\frac{S}{a}+a$.

3. Teng yonli uchburchakning perimetri P ga, asosining uzunligi a ga teng. Uchburchakning yon tomoni uzunligini topish uchun ifoda tuzing.

A) $2a-P$; B) $2P-a$; C) $P-a$; D) $\frac{1}{2}(P-a)$.

4. $a=2{,}5$, $b=2{,}4$ va $c=3{,}5$ bo‘lsa, $V=abc$ ifodaning son qiymatini toping.

A) 18,3; B) 21; C) 2,1; D) 12,1.

5. $a=5$, $b=6{,}4$, $c=4{,}5$ bo‘lsa, $S=2(ab+ac+bc)$ ifodaning son qiymatini toping.

A) 50,45; B) 83,3; C) 166,6; D) 109.

6. Ona farzandlari uchun a so‘mdan 8 ta rasm daftar, b so‘mdan 5 ta ruchka, c so‘mdan 20 ta daftar sotib oldi. Jami xaridni hisoblash uchun ifoda tuzing.

A) $8a+5b+20c$; B) $8a+25\,(b+c)$; C) $800abc$; D) $8a+100ba$.

7. Qavslarni oching va soddalashtiring: $5a+(3a-(4a+3))$.

A) $8a+3$; B) $4a-3$; C) $-4a-3$; D) $3-4a$.

8. Ifodani soddalashtiring va uning $a=2,4;\ b=1,5$ bo‘lgandagi qiymatini toping: $0,5\cdot(2a-3b)-(4b+2,5a)$.

A) 17,4; B) −17,4; C) −1,4; D) −11,85.

9. To‘g‘ri to‘rtburchakning perimetri p ga, asosi a ga teng. Uning balandligini hisoblash uchun ifoda tuzing.

A) $\frac{p-2a}{2}$; B) $2-ap$; C) $\frac{2a-p}{2}$; D) $p-2a$.

10. Ifodani soddalashtiring va uning $a=2,7,\ b=4,2$ bo‘lgandagi son qiymatini toping: $3(2a-b)-2(a-2b)$.

A) 24,36; B) 27,6; C) 8,7; D) 15.

11. Uchburchak bir tomonining uzunligi a ga teng. Ikkinchi tomoni uzunligi bu tomonning 80 % ini tashkil qiladi. Uchinchi tomoni esa birinchi va ikkinchi tomonlar yig‘indisining yarmiga teng bo‘lsa, shu uchburchakning perimetrini toping.

A) $1,8a$; B) $2,7a$; C) $3a$; D) $3a+0,8$.

12. Agar $h=6,\ r=2,\ R=4$ bo‘lsa, $V=\frac{1}{3}\pi h(R^2+Rr+r^2)$ ifodaning son qiymatini toping.

A) 56π; B) 55π; C) 84π; D) 28π.

13. Agar $R=4,5$, va $H=6,5$ bo‘lsa, $S=2\pi\ R(R+H)$ ifodaning son qiymatini toping.

A) 100π; B) 98π; C) 99π; D) $98,5\pi$.

14. Uchburchak bir tomonining uzunligi a ga teng bo‘lib, u ikkinchi tomonidan 2 sm qisqa, uchinchi tomonidan esa 3 sm uzun. Shu uchburchakning perimetrini hisoblash uchun ifoda tuzing.

A) $3a-1$; B) $3a-5$; C) $3a+5$; D) $1-3a$.

Tarixiy ma'lumotlar

Yurtdoshimiz buyuk matematik va astronom olim Abu Abdulloh Muhammad ibn Muso al-Xorazmiy (783—850) ning arifmetik („Algorizmi hind hisobi haqida“) va algebraik („Al-jabr val-muqobala“) asarlari matematikaning rivojiga nihoyatda kuchli ta'sir ko‘rsatdi. Bu asarlar ko‘p tillarga tarjima qilindi, asrlar davomida matematikadan asosiy qo‘llanma bo‘lib xizmat qildi.

„Algorizmi hind hisobi haqida“ risolasining XII asr boshidagi lotincha tarjimasi Angliyaning Kembrij universitetida saqlanadi. Al-Xorazmiyning bu asari tufayli Yevropaga o‘nli sanoq sistemasi kirib borgan.

„Muhammad Muso Xorazmiyning o‘nlik sanoq sistemasini, algoritm va algebra tushunchalarini dunyoda birinchi bo‘lib ilm-fan sohasiga joriy etgani va shu asosda aniq fanlar rivoji uchun o‘z vaqtida mustahkam asos yaratgani umuminsoniy taraqqiyot rivojida qanday katta ahamiyatga ega bo‘lganini barchamiz yaxshi bilamiz“, — deb yozgan edi O‘zbekiston Respublikasining birinchi Prezidenti I. A. Karimov o‘zining „Yuksak ma'naviyat — yengilmas kuch“ asarida.

Xorazmiy algebrasi — „Al-jabr val-muqobala hisobi haqida qisqacha kitob“ asarining arabcha nusxasi Oksford universitetining Bodleyan kutubxonasida saqlanadi. Risola uch qismdan iborat:

1) algebraik qism; 2) geometrik qism; 3) vasiyatlar haqidagi qism (Xorazmiy uni „Vasiyatlar kitobi“ deb atagan). Al-Xorazmiy risolasida barcha masalalarning bayoni va yechimlari so‘zlar bilan beriladi, hech qanday belgilashlar, harfiy ifodalar ishlatilmaydi. Al-Xorazmiy yozadi: „... Men arifmetikaning oddiy va murakkab masalalarini o‘z ichiga oluvchi „Al-jabr val-muqobala hisobi haqida qisqacha kitob“ni ta'lif qildim, chunki meros taqsim qilishda, vasiyatnoma tuzishda, mol taqsimlashda va adliya ishlarida, savdoda va har qanday bitimlarda va shuningdek, yer o‘lchashda, ariqlar o‘tkazishda, muhandislikda va boshqa shunga o‘xshash turlicha ishlarda kishilar uchun bu zarurdir“. Binobarin, olim o‘zining bu asarini kundalik hayot talabi va ehtiyojlarini hisobga olgan holda yozgan.

II BOB
BIR NOMA'LUMLI BIRINCHI DARAJALI TENGLAMALAR

6-§ Tenglama va uning yechimlari

Ushbu masalani yechaylik.

Masala. Qalam va chizg'ich birgalikda 370 so'm turadi. Qalam chizg'ichdan 90 so'm arzon. Chizg'ichning bahosini toping.

△ Aytaylik, chizg'ich x so'm tursin, u holda qalam $(x-90)$ so'm turadi. Masalaning shartiga ko'ra

$$x+(x-90)=370,$$

bundan $2x-90=370$, $2x=460$, $x=230$.

Javob: Chizg'ich 230 so'm turadi. ▲

$x+(x-90)=370$ tenglikda x harfi noma'lum sonni yoki qisqacha noma'lumni bildiradi.

Harf bilan belgilangan noma'lum son qatnashgan tenglik ***tenglama*** *deyiladi.*

Tenglik belgisidan chap va o'ngda turgan ifodalar tenglamaning chap va o'ng qismlari deyiladi. Tenglamaning chap yoki o'ng qismidagi har bir qo'shiluvchi tenglamaning hadi deyiladi.

$2x-90=370$ tenglamada chap qism $2x-90$, o'ng qism esa 370. So'ngra $x=230$ bo'lganda shu tenglamaning chap qismi 370 ga teng, chunki $2\cdot 230-90=370$; o'ng qismi ham 370 ga teng. Demak, $x=230$ bo'lganda bu tenglama to'g'ri tenglikka aylanadi: $2\cdot 230-90=370$. Shu 230 soni berilgan *tenglamaning ildizi* deyiladi.

Tenglamaning ildizi *deb, noma'lumning shu tenglamani to'g'ri tenglikka aylantiradigan qiymatiga aytiladi.*

Masalan, 1 soni

$$2x+3=5$$

tenglamaning ildizi, chunki $2\cdot1+3=5$ — to‘g‘ri tenglik.

Tenglama ikkita, uchta va hokazo ildizlarga ega bo‘lishi mumkin. Masalan,

$$(x-1)(x-2)=0$$

tenglama ikkita ildizga ega: 1 va 2, chunki $x=1$ va $x=2$ da tenglama to‘g‘ri tenglikka aylanadi.

$$(x-3)(x+4)(x-5)=0$$

tenglama esa uchta ildizga ega: 3, -4 va 5.

Tenglama ildizlarining soni cheksiz ko‘p bo‘lishi mumkin. Masalan,

$$2(x-1)=2x-2$$

tenglamaning ildizlari soni cheksiz ko‘p: x ning istalgan qiymati tenglamaning ildizi bo‘ladi, chunki har bir x da tenglamaning chap qismi o‘ng qismiga teng.

Tenglama ildizlarga ega bo‘lmasligi ham mumkin. Masalan, $2x+5=2x+3$ tenglamaning ildizlari yo‘q, chunki x ning istalgan qiymatida bu tenglamaning chap qismi o‘ng qismidan katta bo‘ladi.

Tenglamani yechish — uning barcha ildizlarini topish yoki ularning yo‘qligini ko‘rsatish demakdir.

Sodda hollarda x ning tenglamaning ildizi bo‘ladigan qiymatini tanlash oson bo‘ladi. Masalan, $2x+1=3$ tenglamaning ildizi 1 soni ekanligini osongina ko‘rish mumkin. Biroq murakkab holda ildizni birdaniga topish oson bo‘lmaydi. Masalan,

$$\frac{x-6}{5}+\frac{4(x+3)}{2}-1=\frac{x-1}{2}+3x-\frac{7x-1}{10}$$

tenglama $x=7$ bo‘lganda to‘g‘ri tenglikka aylanishini bilish ancha qiyin. Shuning uchun tenglamalarni yechishni o‘rganish muhim.

Ko‘pgina amaliy masalalarni yechish

$$ax = b \qquad (1)$$

ko‘rinishdagi tenglamaga keltiriladi, bunda a va b — berilgan sonlar, x — noma’lum son. (1) tenglama *chiziqli tenglama* deb ataladi.

Masalan, $3x = 1$, $-2x = 3$, $\frac{3}{5}x = -\frac{1}{2}$ — chiziqli tenglamalardir.

Mashqlar

79. Tenglik shaklida yozing:

1) 34 soni x sondan 18 ta ortiq;
2) 56 soni 14 sonidan x marta ortiq;
3) x va 3 sonlari ayirmasining ikkilangani 4 ga teng;
4) x va 5 sonlari yig‘indisining yarmi ularning ko‘paytmasiga teng.

80. 3; -2; 1 sonlaridan qaysi biri tenglamaning ildizi bo‘ladi:

1) $3x = -6$;
2) $x + 3 = 6$;
3) $4x - 4 = x + 5$;
4) $5x - 8 = 2x + 4$?

81. (Og‘zaki.) x ning qanday qiymatlarida tenglama to‘g‘ri tenglikka aylanadi:

1) $x + 5 = -6$; 2) $4 - x = -1$; 3) $2x - 1 = 0$; 4) $3x + 2 = 0$?

82. -1; $\frac{1}{2}$; 1 sonlari orasida tenglamaning ildizi bormi:

1) $4(x - 1) = 2x - 3$;
2) $7(x + 1) - 6x = 10$;
3) $3(x + 2) = 4 + 2x$;
4) $5(x + 1) - 4x = 4$?

83. Ildizi:

1) 5 soni; 2) 3 soni; 3) -6 soni; 4) -4 soni bo‘lgan tenglama tuzing.

84. a sonni shunday tanlangki, $4x - 3 = 2x + a$ tenglama

1) $x = 1$; 2) $x = -1$; 3) $x = \frac{1}{2}$; 4) $x = 0{,}3$

ildizga ega bo‘lsin.

7-§ Bir noma'lumli birinchi darajali tenglamalarni yechish

Al-Xorazmiyning „Kitob al-muxtasar fi hisob al-jabr val-muqobala“ asaridagi al-jabr musbat hadlarni tiklash, ya'ni manfiy hadlarni tenglamaning bir qismidan ikkinchi qismiga musbat qilib o'tkazishni, val-muqobala esa tenglamaning ikkala qismidan teng hadlarni tashlab yuborishni bildirgan.

Bu bir noma'lumli tenglamalarni yechish to'g'ri tengliklarning sizlarga ma'lum xossalariga asoslangan ekanini ko'rsatadi. Shu xossalarni eslatib o'tamiz.

Xossaning so'z bilan ifodalanishi	Xossaning umumiy ko'rinishda yozilishi	Misol
1. Agar to'g'ri tenglikning ikkala qismiga bir xil son qo'shilsa yoki ikkala qismidan bir xil son ayrilsa, u holda yana to'g'ri tenglik hosil bo'ladi.	Agar $a = b$ bo'lib, l ixtiyoriy son bo'lsa, u holda $a + l = b + l$, $a - l = b - l$ bo'ladi.	$7 = 7$ $7 + 2 = 7 + 2$ $7 - 2 = 7 - 2$
2. Agar to'g'ri tenglikning ikkala qismi nolga teng bo'lmagan ayni bir songa ko'paytirilsa yoki bo'linsa, u holda yana to'g'ri tenglik hosil bo'ladi.	Agar $a = b$ bo'lib, $m \neq 0$ bo'lsa, u holda $a \cdot m = b \cdot m$ va $a : m = b : m$ bo'ladi.	$27 = 27$ $27 \cdot 3 = 27 \cdot 3$ $27 : 3 = 27 : 3$

Birinchi xossadan qo'shiluvchilarni, ularning ishoralarini qarama-qarshisiga almashtirib, tenglikning bir qismidan ikkinchi qismiga olib o'tish mumkinligi kelib chiqadi.

○ Aytaylik, $a = b + m$ bo'lsin, u holda

$$a + (-m) = b + m + (-m);\ \ a - m = b.$$ ●

Tengliklarning bu xossalari tenglamalarni yechishda qanday qo'llanishini ko'raylik.

1-masala. $9x - 23 = 5x - 11$ tenglamani yeching.

△ x son berilgan tenglamaning ildizi, ya'ni x shunday sonki, uni tenglamaga qo'yilganda tenglama to'g'ri tenglikka aylanadi, deb faraz qilamiz.

Noma’lum qatnashgan $5x$ hadni „–“ ishora bilan tenglikning chap qismiga, –23 hadni „+“ ishora bilan o‘ng qismiga olib o‘tamiz.

Natijada, yana to‘g‘ri tenglik hosil bo‘ladi:

$$9x-5x=23-11.$$

Tenglamaning ikkala qismidagi o‘xshash hadlarni ixchamlab,

$$4x=12$$

tenglamani hosil qilamiz. Bu tenglamaning ikkala qismini 4 ga bo‘lib, $x = 3$ ekanini topamiz.

Shunday qilib, tenglama ildizga ega deb faraz qilib, bu ildiz faqat 3 soniga teng bo‘lishi mumkinligini ko‘rdik. $x = 3$ haqiqatan ham berilgan tenglamaning ildizi bo‘lishini tekshiramiz: $9 \cdot 3 - 23 = 5 \cdot 3 - 11$. Bu to‘g‘ri tenglik, chunki uning chap va o‘ng qismlari ayni bir songa – 4 soniga teng.

Demak, berilgan tenglama faqat bitta ildizga ega: $x = 3$. ▲

Tekshirishni bajarmaslik ham mumkinligini ta’kidlaymiz, chunki tenglikning foydalanilgan xossalari bir to‘g‘ri tenglikni ikkinchi to‘g‘ri tenglik bilan almashtirishga imkon beradi. Yechishning bu usulida har doim to‘g‘ri natija hosil qilinadi (agar hisoblashlarda xatolikka yo‘l qo‘yilmasa, albatta).

$-3x$ $5x - 7 = 3x + 11$ $+7$ $5x - 3x = 11 + 7$	**AL-JABR:** $3x$, chapga $-3x$ bo‘lib o‘tasan! –7, sen o‘ngga +7 bo‘lib o‘tasan!

$\cancel{4x} - \cancel{5} + 2x = \cancel{4x} + 8 - \cancel{5}$ $2x = 8$	**VAL-MUQOBALA:** chap va o‘ng qismdagi – 5 lar-u, $4x$ lar, sizlar bilan xayrlashamiz!

Tenglama yechilishini yozishda 1- masalani yechishdagidek batafsil yozma tushuntirishlarni bajarish shart emas.

Masalan, $5x+17=2x+5$ tenglamaning yechilishini bunday yozish mumkin:

$$5x-2x=5-17,\ \ 3x=-12,\ \ x=-4.$$

Javob: $x=-4$.

2- masala. $2(x+3)-3(x+2)=5-4(x+1)$ tenglamani yeching.

△ Tenglamaning chap va o‘ng qismlarini soddalashtiramiz: qavslarni ochamiz va o‘xshash hadlarni ixchamlaymiz. Natijada, $2x+6-3x-6=5-4x-4$, $-x=-4x+1$ tenglamani hosil qilamiz.

Demak, $3x=1$, bundan $x=\frac{1}{3}$. Javob: $x=\frac{1}{3}$. ▲

3- masala. $\frac{5x}{2}-\frac{x-3}{3}=1+\frac{x-5}{6}$ tenglamani yeching.

△ Tenglamaning ikkala qismini kasrlarning umumiy maxrajiga, ya’ni 6 ga ko‘paytiramiz, u holda

$$\frac{5x}{2}\cdot 6-\frac{x-3}{3}\cdot 6=1\cdot 6+\frac{x-5}{6}\cdot 6; \qquad 15x-2(x-3)=6+(x-5).$$

Qavslarni ochamiz va o‘xshash hadlarni ixchamlaymiz:

$$15x-2x+6=6+x-5; \qquad 13x+6=x+1,$$

bundan $12x=-5$, $x=-\frac{5}{12}$. Javob: $x=-\frac{5}{12}$. ▲

Shunday qilib, tenglamani yechishda *tenglamaning* quyidagi *asosiy xossalaridan* foydalaniladi.

1- xossa. *Tenglamaning istalgan hadi ishorasini qarama-qarshisiga o‘zgartirib, uning bir qismidan ikkinchi qismiga o‘tkazish mumkin.*

2- xossa. *Tenglamaning ikkala qismini nolga teng bo‘lmagan bir xil songa ko‘paytirish yoki bo‘lish mumkin.*

Bu xossalar bir noma’lumli istalgan tenglamani yechish imkonini beradi. Buning uchun:

1) noma’lum qatnashgan hadlarni tenglikning chap

qismiga, noma'lum qatnashmagan hadlarni esa o'ng qismiga o'tkazish lozim (1- xossa);

2) o'xshash hadlarni ixchamlash kerak;

3) tenglamaning ikkala qismini noma'lum oldida turgan koeffitsiyentga (agar u nolga teng bo'lmasa) bo'lish (2- xossa) kerak.

Ko'rib chiqilgan misollarda har bir tenglama bitta ildizga ega bo'ldi. Ammo ba'zi hollarda bir noma'lumli tenglama ildizlarga ega bo'lmasligi mumkin yoki cheksiz ko'p ildizga ega bo'lishi mumkin. Shunday tenglamalarga misollar keltiramiz.

4- masala. $2(x+1)-1=3-(1-2x)$ tenglama ildizlarga ega emasligini ko'rsating.

△ Tenglamaning ikkala qismini soddalashtiramiz:

$$2x+2-1=3-1+2x,\ \ 2x+1=2+2x,$$

bundan

$$2x-2x=2-1,\ \ 0\cdot x=1.$$

Bu tenglama ildizlarga ega emas, chunki uning $0\cdot x$ dan iborat chap qismi nolga teng, o'ng qismi esa 1 ga teng, ammo $0\neq 1$.

Javob: tenglama yechimga ega emas. ▲

5- masala. $3(1-x)+2=5-3x$ tenglama cheksiz ko'p yechimga ega ekanligini ko'rsating.

△ Tenglamani soddalashtiramiz:

$3-3x+2=5-3x;\ \ 5-3x=5-3x;\ \ -3x+3x=5-5,\ \ 0\cdot x=0$

Demak, x ning istalgan qiymati bu tenglamaning ildizi bo'ladi.

Javob: tenglama cheksiz ko'p yechimga ega.▲

Mashqlar

Tenglamani yeching **(85 — 96)**:

85. 1) $11x=50$; | 2) $-9x=243$; | 3) $4x=0,24$; | 4) $7x=7,063$.

86. 1) $9x=\frac{2}{5}$; | 2) $3x=2\frac{1}{7}$; | 3) $\frac{1}{2}x=3$; | 4) $\frac{3}{4}x=\frac{1}{2}$.

87. 1) $0,3x=6$; | 2) $1,3x=-1,69$; | 3) $0,7x=49$; | 4) $10x=0,5$.

88. 1) $8x=8$; | 2) $\frac{1}{4}x=16$; | 3) $3^2x=243$; | 4) $16x=16$.

89. 1) $5x=\left(\frac{5}{7}\right)^2$; | 2) $4x=-\left(\frac{4}{5}\right)^2$; | 3) $-0,1x=10^3$; | 4) $0,3x=-10^2$.

90. 1) $25x-1=9$;
2) $7x+8=11$;
3) $3x-5=10-x$;
4) $4x+4=x+5$.

91. 1) $5x+3(3x+7)=35$;
2) $8x-(7x+8)=9$;
3) $8y-9-4y+5=12y-4-5y$;
4) $4+8y+8=2y-10-7y+9$.

92. 1) $\frac{11}{7}=\frac{2-x}{5}$; 2) $\frac{3x}{5}=\frac{6+x}{3}$; 3) $\frac{x}{3}+\frac{x}{5}=8$; 4) $\frac{y}{3}+\frac{y}{4}=14$.

93. 1) $3y+5=4\left(9-\frac{y}{2}\right)$;
2) $8\left(11-\frac{3}{4}z\right)=16z-44$;
3) $3\left(5+\frac{x}{2}\right)=4+2x$;
4) $2\left(3-\frac{x}{3}\right)=5+x$.

94. 1) $0,71x+1,98=0,37x-1,76$;
2) $0,18y-7,4=0,05y-5,71$;
3) $5(5x-1)-2,7x+0,2x=6,5-0,5x$;
4) $0,36x-0,6=0,3(0,4x-1,2)$.

95. 1) $11\frac{2}{3}x-5\frac{1}{6}=3\frac{3}{4}+2\frac{3}{4}x$;
2) $12\frac{3}{4}+\frac{3}{7}y=\frac{y}{2}-10\frac{1}{28}$;
3) $\frac{6x+7}{7}=3-\frac{5x-3}{8}$;
4) $10-\frac{3x-1}{2}=\frac{6x+3}{11}$.

96. 1) $\frac{4x-51}{3}-\frac{17-3x}{4}=\frac{x+5}{2}$;
2) $\frac{3x-7}{4}-\frac{9x+11}{8}=\frac{3-x}{2}$;
3) $\frac{9x-5}{2}-\frac{3+5x}{3}-\frac{8x-2}{4}=2$;
4) $\frac{4x-3}{2}-\frac{5-2x}{3}=\frac{3x-4}{3}$.

№ 3 — *Buvijon, nabirangiz necha yoshda?*
— *Mening yoshim nechada bo‘lsa, nabiram shuncha oylik.*
— *Buvijon, sizning yoshingiz nechada?*
— *Nabiram yoshi bilan mening yoshimni qo‘shsang, 65 chiqadi. Nabiramning yoshini endi o‘zing topa qol.*

97. Tenglamaning ildizlarga ega emasligini ko‘rsating:

1) $28 - 20x = 2x + 25 - 16x - 12 - 6x$;

2) $25x - 17 = 4x - 5 - 13x + 14 + 34x$;

3) $\frac{x-1}{3} + \frac{5x+2}{12} = \frac{5+3x}{4}$;

4) $\frac{2x+1}{3} - \frac{7x+5}{15} = \frac{x-2}{5}$.

98. x ning istalgan qiymati tenglamaning ildizi bo‘la olishini ko‘rsating:

1) $10-4x+3=9x-2-6x+9-7x+6$;

2) $9x+4-5x=8+7x-9-3x+5$;

3) $6(1,2x-0,5)-1,3x=5,9x-3$;

4) $8(1,3x+0,25)-6,6x=3,8x+2$.

99. Tenglamani yeching:

1) $3(x-1)-2(x+2)=4x+8$;

2) $4(x+1,5)+3(1-x)=10$;

3) $4(3x+2)-7(x+1)=3(x-1)$;

4) $2,5(2x+3)-2(x+2,5)=3,5+2x$.

100. Tenglamani yeching:

1) $\frac{96}{7,2} = \frac{4x+300}{21}$;

2) $\frac{3x+14,7}{20,4} = \frac{7,5}{10}$;

3) $4,2 : (2x-7) = 10 : 7\frac{1}{7}$;

4) $4\frac{1}{11} : 10 = 4,5 : (3x-1)$.

8- §. Masalalarni tenglamalar yordamida yechish

Tenglamalarni qo‘llash ko‘pgina masalalarni yechishni osonlashtiradi. Bunda masalani yechish, odatda, ikki bosqichdan iborat bo‘ladi:

1) masalaning sharti bo‘yicha tenglama tuzish;
2) hosil bo‘lgan tenglamani yechish.

Ushbu masalani yechaylik.

Masala. Sayyohlar tushgan kema sohildagi bekatdan daryo oqimi bo‘yicha jo‘nab, 5 soatdan keyin qaytib kelishi kerak. Daryo oqimining tezligi 3 km/soat; kemaning turg‘un suvdagi tezligi 18 km/soat. Agar sayyohlar qaytishdan oldin qirg‘oqda 3 soat dam olgan bo‘lsalar, ular sohildagi bekatdan qancha masofaga suzib borganlar?

△ 1) Izlanayotgan masofa x kilometr bo‘lsin. Kema bu masofani oqim bo‘yicha $18 + 3 = 21$ (km/soat) tezlik bilan o‘tadi va bunga $\frac{x}{21}$ soat sarf qiladi. Kema $18 - 3 = 15$ (km/soat) tezlik bilan orqasiga qaytadi va bunga $\frac{x}{15}$ soat sarf qiladi. Sayyohlar qirg‘oqda 3 soat dam oldilar. Demak, sayohat $\left(\frac{x}{21}+\frac{x}{15}+3\right)$ soat davom etdi, bu esa masala shartiga ko‘ra 5 soatga teng. Shunday qilib, biz noma’lum x masofani aniqlash uchun quyidagi tenglamani hosil qildik:

$$\frac{x}{21}+\frac{x}{15}+3=5;$$

2) endi

$$\frac{x}{21}+\frac{x}{15}=2$$

tenglamani yechamiz. Bu tenglamaning ikkala qismini 105 ga (21 va 15 sonlarining eng kichik umumiy bo‘linuvchisiga) ko‘paytirib, $5x + 7x = 210$, $12x = 210$ tenglikni hosil qilamiz, bundan $x = 17{,}5$.

Javob: kema sohildagi bekatdan 17,5 km masofaga suzib boradi. ▲

Masalani yechishning birinchi bosqichida (ya'ni tenglama tuzishda) kema bilan daryo oqimi tezliklari oqim bo'yicha harakatda qo'shilishi, oqimga qarshi harakatda esa ayirilishi va yo'lning tezlikka nisbati harakat vaqti ekanligini bilish zarur bo'ldi.

Ikkinchi bosqichda (ya'ni hosil bo'lgan tenglamani yechishda) tenglamalarning bundan oldingi paragrafda o'rganilgan xossalarini qo'llash talab etildi.

Matnli masala mazmuniga mos tenglama tuzish – masala shartini „matematika tili" ga o'tkazish – masalaning matematik modelini tuzish demakdir. Bitta masalani hal qilish uchun turli tenglama, turli matematik model tuzish mumkin.

Mashqlar

101. A va B shaharlari orasidagi masofa 256 km. A dan B ga qarab 66 km/soat tezlik bilan yuk poyezdi yo'lga chiqdi. Oradan 20 minut o'tgach, B dan A ga qarab 90 km/soat tezlik bilan tezyurar poyezd yo'lga chiqdi. Yuk poyezdi yo'lga chiqqanidan qancha vaqtdan so'ng tezyurar poyezd bilan uchrashadi:

Bu masalani yechish uchun tenglamalarni quyidagicha tuzish mumkin:

a) $66x+90\left(x-\frac{1}{3}\right)=256;$

b) $256-66\cdot\frac{1}{3}=(66+90)\cdot\left(x-\frac{1}{3}\right);$

d) $\frac{x}{66}-\frac{256-x}{90}=\frac{1}{3};$

e) $256-90x=66\cdot\left(x+\frac{1}{3}\right).$

1) Har bir tenglamada x nimani bildiradi?

2) Har bir tenglamada qanday miqdorlar tenglashtirilgan?

102. 1) Belgilangan ishni 15 kishi 12 kunda bajarishi mumkin. 4 kun ishlagandan so'ng, beshinchi kuni ularga yordam

berish uchun 5 kishi kelib qo‘shildi. Qolgan ish necha kunda tugatilgan?

2) Ishchilar belgilangan vazifani 15 kunda bajara olishadi, 5 kundan so‘ng ularga yana 8 kishi qo‘shildi va birgalikda qolgan ishni 6 kunda tugallashdi. Ishchilar dastlab necha kishi edilar?

3) Bir ishni 10 kishi 8 kunda bajara oladi. 2 kundan so‘ng (uchinchi kuni) ularga yordam berish uchun bir nechta kishi kelib qo‘shildi va qolgan ish 4 kunda bajarildi. Nechta kishi kelib qo‘shilgan?

103. 1) Uchta firmada 624 nafar ishchi bor. Ikkinchi firmada birinchisidagiga qaraganda ishchilar 5 marta ko‘p, uchinchi firmada esa birinchi va ikkinchi firmalarda birgalikda nechta ishchi bo‘lsa, shuncha ishchi bor. Har bir firmada nechtadan ishchi bor?

2) Uchta kichik korxonada 792 ta mahsulot tayyorlandi. Ikkinchi kichik korxonada birinchi kichik korxonaga qaraganda 3 marta ko‘p, uchinchi kichik korxonada esa ikkinchisidagidan 2 marta kam mahsulot tayyorlandi. Har bir kichik korxonada nechtadan mahsulot tayyorlangan?

104. 1) Teng yonli uchburchakning perimetri 25 sm ga teng. Agar uning yon tomoni asosidan 5 sm ortiq bo‘lsa, uchburchak tomonlari uzunliklarini toping.

2) Teng yonli uchburchakda asos yon tomonning $\frac{3}{4}$ qismini tashkil etadi. Agar uchburchakning perimetri 22 sm ga teng bo‘lsa, uning tomonlari uzunliklarini toping.

105. 1) Eni 200 m bo‘lgan to‘g‘ri to‘rtburchak shaklidagi maydonning chegarasi bo‘ylab ariq qazildi. Ariqning uzunligi 1 km. Maydonning bo‘yini toping.

2) Bo‘yi enidan 2 marta uzun bo‘lgan to‘g‘ri to‘rtburchak shaklidagi maydonni uzunligi 120 m bo‘lgan panjara bilan o‘rashdi. Maydonning bo‘yi va enini toping.

106. Yig‘indisi 81 ga teng bo‘lgan uchta ketma-ket toq sonni toping.

107. To‘rtta ketma-ket juft son berilgan. Agar chetki sonlar yig‘indisining ikkilanganidan o‘rtadagi sonlar musbat ayirmasining uchlangani ayirilsa, 22 hosil bo‘ladi. Shu sonlarni toping.

108. 1) Yangi moslamani ishga tushirgach, ustaning mo‘ljallangan ishni bajarishga ketadigan vaqti 20 % ga kamaydi. Uning mehnat unumdorligi necha foizga oshgan?

2) Fabrikaga avtomat o‘rnatildi. U bir soatda ishchiga qaraganda 8 ta ortiq mahsulot ishlab chiqaradi. 2 soatdan keyin avtomat ishchining 6 soatlik rejasini bajardi. Avtomat bir soatda nechta mahsulot ishlab chiqaradi?

3) Ustaning mehnat unumdorligi 20 % ga ortsa, uning ish rejasini bajarishga ketadigan vaqti necha foizga qisqaradi?

109. Uzunligi 27 m bo‘lgan mis simni massasi va ko‘ndalang kesimi mis simnikidek bo‘lgan alumin sim bilan almashtirishmoqchi. Nima deb o‘ylaysiz, alumin simning uzunligi necha metr bo‘larkin?

110. Bir nechta do‘kon olma solingan 175 ta yashikni teng bo‘lib olishmoqchi edi. Ammo 2 ta do‘kon olmalarni olmasligini bildirdi. Natijada, qolgan har bir do‘konga mo‘ljaldagidan 10 yashik olma ortiqcha berildi. Do‘konlar nechta ekan?

111. 1) Idishda qanchadir litr suv bor. Agar idishga 3 *l* suv quyilsa, idishning yarmi to‘ladi. Agar 3 *l* suv to‘kib tashlansa, qolgan suv idishning $\frac{1}{8}$ qismini egallaydi. Dastlab idishda necha litr suv bo‘lgan?

2) Idishning ichidagi suv bilan birgalikdagi massasi 12 kg ga teng. Idishdagi suvning $\frac{3}{5}$ qismi gullarga quyilgach, idishning massasi ichidagi suv massasidan 2 marta kamligi aniqlandi. Idishning massasi necha kilogramm ekan?

112. 1) Neft omborida 6340 t benzin bor edi. Ikkinchi kuni ombor birinchi kundagidan 423 t ko‘p, uchinchi kuni esa

ikkinchi kundagidan 204 t kam benzin tarqatdi. Shundan so‘ng omborda 3196 t benzin qoldi. Ombor birinchi kuni necha tonna benzin tarqatgan?

2) Do‘konda uch kunda 110 kg yog‘ sotildi. Ikkinchi kuni birinchi kundagining 37,5 % qismicha, uchinchi kuni esa dastlabki ikki kunda qancha yog‘ sotilgan bo‘lsa, shuncha sotildi. Do‘konda birinchi kuni necha kilogramm yog‘ sotilgan?

113. 1) Usta va o‘g‘li buyurtmani 10 kunda bajarishlari kerak edi. Ular yangi moslamani ishlatib, har kuni rejadan tashqari 27 ta mahsulot tayyorlab, 7 kunda topshiriqni bajaribgina qolmasdan, balki ortiqcha yana 54 ta mahsulot tayyorlashdi. Usta va o‘g‘li bir kunda nechta mahsulot tayyorlashgan?

2) Zavod mashina ishlab chiqarish bo‘yicha buyurtmani 15 kunda bajarishi kerak edi. Zavod yangi texnologiyani joriy etib, har kuni rejadan tashqari 2 ta ortiq mashina ishlab chiqarib, muddatga 2 kun qolganda faqat rejani bajaribgina qolmasdan, rejadan ortiq yana 6 ta mashina ishlab chiqardi. Zavod 15 kunda reja bo‘yicha nechta mashina ishlab chiqarishi kerak edi?

O‘zingizni tekshirib ko‘ring!

1. 1; 0; –4 sonlari ichida $3(x-7)+4=7x-1$ tenglamaning ildizi bormi?

2. Tenglamani yeching:

1) $2x-3(x-1)=4+2(x-1)$;

2) $\frac{x}{3}+\frac{x+1}{4}=2$.

3. Sotuvchi molining 20 % ini 40 % foyda bilan sotdi. Jami sotuvdan 32 % foyda ko‘rish uchun u qolgan molini necha foiz foyda bilan sotishi kerak?

114. 1) 1 kg i 200 so‘mdan olingan uzumning 3 kg idan 1 kg sharbat olinib, 720 so‘mga sotildi. Uzumning narxi 50 so‘mga arzonlashdi. Tijoratchi avvalgi foydani saqlab qolmoqchi. Sharbatning yangi narxi dastlabkisidan necha so‘mga arzon qilinishi kerak?
2) 20 % li sharbat hosil qilmoqchisiz. Ayting-chi, necha litr qaynagan suvga 200 gramm shakar qo‘shasiz?

115. 1) Idishda dastlab ma’lum miqdor suv bor edi. Agar idishga a litr suv quyilsa, idishning $\frac{1}{8}$ qismi to‘ladi. Agar idishdagi dastlabki suvdan a litr olib tashlansa, idishning $\frac{3}{20}$ qismi to‘la bo‘ladi. Dastlab idishning qancha qismi to‘la bo‘lgan?

2) Idishning $\frac{1}{5}$ qismi bo‘sh. Ahmad idishni to‘ldirmoqchi. U idishdagi suvning qancha qismi qadar suv quysa, idish to‘ladi? Unga yordam bering.

116. Yerning birinchi ikkita sun’iy yo‘ldoshi massasi 592,4 kg ni tashkil qildi. Birinchi sun’iy yo‘ldosh uchinchisidan 1243,4 kg yengil, ikkinchisi esa 818,2 kg yengil. Yerning birinchi uchta sun’iy yo‘ldoshining har birining massasini toping.

117. Qayiq daryo oqimi bo‘yicha 2,4 soat va oqimga qarshi 3,2 soat suzdi. Qayiqning oqim bo‘yicha bosib o‘tgan yo‘li oqimga qarshi bosib o‘tgan yo‘lidan 13,2 km ortiq bo‘ldi. Agar daryo oqimining tezligi 3,5 km/soat bo‘lsa, qayiqning turg‘un suvdagi tezligini toping.

118. Bo‘ston va Guliston qishloqlari orasidagi masofa 72 km. Bu qishloqlardan ikkita sayyoh bir vaqtda yo‘lga chiqdi. Birining tezligi soatiga v kilometr, ikkinchisiniki esa soatiga u kilometr. 2 soatdan so‘ng ular orasidagi masofa necha kilometr bo‘ladi? Hamma hollarni qarang va tahlil qiling.

№ 4 *Xodani 3 bo'lakka arralash uchun 12 minut kerak. Shu xodani 4 bo'lakka arralash uchun necha minut kerak bo'ladi?*

119. Idishning $\frac{1}{3}$ qismi suv bilan to'la. Bu suvning $\frac{1}{4}$ qismi ishlatilgandan keyin unga 45 *l* suv solinsa, idishning $\frac{1}{8}$ qismi bo'sh bo'ladi. Idishga jami necha litr suv ketadi?

120. Sinovda 60 ta savol berildi, har bir to'g'ri javob 5 ballga baholandi. 4 ta noto'g'ri javob uchun jarima sifatida bitta to'g'ri javob bekor qilinadi. Bu sinovda hamma savollarni belgilagan bir o'quvchi 225 ball olgan bo'lsa, u nechta savolga to'g'ri javob bergan?

II bobga doir sinov mashqlari — testlar

1. $\frac{5x-3}{8}=\frac{x}{2}+3+\frac{11-3x}{4}$ tenglamaning ildizi x_0 bo'lsa, x_0^2+1 ifodaning son qiymatini toping.

A) 50; B) 10; C) 5; D) 37.

2. $\frac{2x+1}{3}+2=\frac{3x-2}{2}+\frac{x+1}{3}$ tenglamaning ildizi x_0 bo'lsa, $18:x_0$ ifodani hisoblang:

A) 6; B) 7; C) -7; D) $46\frac{2}{7}$.

3. $(x+3):(x-2)=5:3$ tenglamaning ildizi x_0 bo'lsa, $2x_0+61$ ifodaning son qiymatini toping.

A) -80; B) 70; C) 80; D) 81.

4. $4:(2x+5)=2:(3x-2)$ tenglamaning ildizi x_0 bo'lsa, $4x_0+11$ ifodaning son qiymatini toping.

A) -18; B) -20; C) 19; D) 20.

5. $0{,}8\cdot(1{,}5x-2)-0{,}4x=0{,}3\cdot(6x-5)-2{,}6$ tenglamaning ildizi x_0 bo'lsa, $x_0^2-0{,}5x_0$ ifodaning son qiymatini toping.

A) 5; B) 1,25; C) 6,25; D) -5.

6. Uchta javonda hammasi bo‘lib 385 ta kitob bor. Birinchi javonda ikkinchisiga qaraganda 8 ta ko‘p, ammo uchinchi javondagidan 9 ta kam kitob bor. Har bir javonda nechtadan kitob bor?

A) 128; 120; 137; B) 127; 119; 139;
C) 127; 122; 136; D) 126; 134; 125.

7. Teng yonli uchburchakning perimetri 51 sm ga teng. Asos yon tomondan 6 sm uzun. Shu uchburchak yon tomonining asosiga nisbatini toping.

A) 7 : 5; B) 5 : 7; C) 2 : 3; D) 10 : 7.

8. Teng yonli uchburchakning perimetri 42 sm ga teng. Yon tomon asosning $\frac{2}{3}$ qismini tashkil qiladi. Shu uchburchakning asosi yon tomonidan necha santimetr uzun?

A) 7,5 sm. B) 6,5 sm; C) 6 sm; D) 7 sm.

9. Usta buyurtmani 8 kunda bajarishi kerak edi. U har kuni rejadan tashqari 6 ta mahsulot tayyorlab, buyurtmani 5 kunda bajaribgina qolmasdan, balki ortiqcha yana 12 ta mahsulot tayyorladi. Usta reja bo‘yicha bir kunda nechta mahsulot tayyorlashi kerak edi?

A) 6; B) 4; C) 5; D) 7.

Tenglamani yeching **(10—11):**

10. $8(x+2)-5x=-2(x+4{,}5)$.

A) -5; B) 5; C) 6; D) $-4{,}5$.

11. $6\cdot(2{,}3x-1)-3{,}5x+0{,}7x=0{,}5(x-14)$.

A) $-\frac{2}{21}$; B) 10,5; C) $\frac{2}{21}$; D) 7.

12. Uchburchakning bir tomoni ikkinchi tomonidan 3 sm uzun, uchinchi tomonidan esa 5 sm qisqa. Agar uchburchakning perimetri 41 sm bo‘lsa, uning eng uzun tomoni eng qisqa tomonidan necha marta uzun?

A) 2; B) 1,5; C) 1,3; D) 1,8.

13. Birinchi to‘pda 75 m, ikkinchi to‘pda 120 m atlas bor edi. Ikkinchi to‘pdan birinchidan sotilganiga qaraganda 3 marta ko‘p atlas sotildi. Natijada birinchi to‘pda ikkinchisiga qaraganda 2 marta ko‘p atlas qoldi. Har bir to‘pdan necha metrdan atlas sotilgan?

A) 24 m; 72 m; B) 30 m; 90 m; C) 15 m; 45 sm; D) 33 m; 99 m.

14. Tenglamani yeching:

$3(x+2)-2(x+3)=7-5(x+1)$.

A) $-\frac{1}{3}$; B) $\frac{1}{3}$; C) -1; D) 2.

Tarixiy ma’lumotlar

Muhammad ibn Muso al-Xorazmiy „Al-jabr val-muqobala hisobi haqida qisqacha kitob“ asarida kiritilgan „al-jabr“, „val-muqobala“ qoidalarini biz 7-§ da tenglamaning asosiy xossalari sifatida bayon qildik.

Algebrada uch xil sonlar bilan ish ko‘riladi, deydi al-Xorazmiy. Ular:

— ildiz yoki narsa (tenglamadagi noma’lum son x);

— kvadrat (mol) (noma’lumning kvadrati — x^2);

— oddiy son (bunda natural son nazarda tutiladi).

Xorazmiy shu uch xil miqdorlar orasidagi bog‘lanishlarni tahlil qiladi va quyidagi tenglamalarni yechish usullarini ko‘rsatadi:

1) $cx^2=bx$ — kvadratlar ildizlarga teng;
2) $cx^2=a$ — kvadratlar sonlarga teng;
3) $bx=a$ — ildizlar songa teng;
4) $cx^2+bx=a$ — kvadratlar va ildizlar sonlarga teng;
5) $cx^2+a=bx$ — kvadratlar va son ildizlarga teng;
6) $bx+a=cx^2$ — ildizlar va son kvadratlarga teng.

Biz 7-sinfda faqat chiziqli tenglamalarni o‘rganamiz [3) banddagi $bx=a$ tenglama]. Qolganlari 8-sinfda o‘rganiladi. Har qanday chiziqli yoki kvadrat tenglama „al-jabr“, „val-muqobala“ almashtirishlari natijasida yuqoridagi 6 ta tenglamaning biriga keltirilishi mumkin.

III BOB
BIRHADLAR VA KO‘PHADLAR

9-§ Natural ko‘rsatkichli daraja

Teng sonlarni qo‘shishni ko‘paytirish bilan almashtirish mumkin:

$$\underbrace{3+3+3+3+3}_{5 \text{ marta}} = 3\cdot 5 \qquad \underbrace{a+a+a+a+\ldots+a}_{n \text{ marta}} = a\cdot n$$

Bir xil sonlarning ko‘paytmasini ham ko‘p hollarda ixchamroq yozuv bilan almashtirish maqsadga muvofiq bo‘ladi. Tomonining uzunligi 5 birlikka teng kvadratni qaraylik (6- rasm). U $5 \cdot 5 = 25$ ta birlik kvadratdan iborat. Tomonining uzunligi 5 birlikka teng kub (7- rasm) esa $5 \cdot 5 \cdot 5 = 125$ ta birlik kubni o‘z ichiga oladi.

Sizga ma’lumki, $5 \cdot 5$ ko‘paytma 5^2 (o‘qilishi: „beshning kvadrati“); $5 \cdot 5 \cdot 5$ ko‘paytma esa 5^3 (o‘qilishi: „beshning kubi“) kabi belgilanadi:

$$5 \cdot 5 = 5^2, \quad 5 \cdot 5 \cdot 5 = 5^3.$$

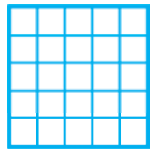
Xuddi shu kabi, ko‘paytuvchilari bir xil sonlardan iborat ko‘paytmani yangi amal — *darajaga ko‘tarish* amali bilan almashtirish mumkin:

$$\underbrace{3\cdot 3\cdot 3\cdot 3\cdot 3}_{5 \text{ marta}} = 3^5, \qquad \underbrace{\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\ldots\cdot\frac{1}{7}}_{9 \text{ marta}} = \left(\frac{1}{7}\right)^9,$$

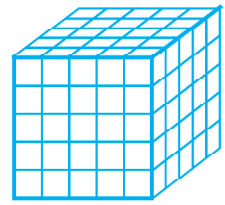
$$0{,}4 = (0{,}4)^1.$$

Umuman, n ta teng ko‘paytuvchining ko‘paytmasini belgilash uchun a^n yozuvidan foydalaniladi:

$$\underbrace{a\cdot a\cdot a\cdot a\cdot\ldots\cdot a}_{n \text{ marta}} = a^n.$$



6- rasm.



7- rasm.

U bunday o‘qiladi: „a sonning n ko‘rsatkichli darajasi“. Odatda, qisqacha qilib: „a ning n- darajasi“ deb aytiladi.

a sonning n natural ko‘rsatkichli darajasi deb, har biri a ga teng bo‘lgan n ta ko‘paytuvchining ko‘paytmasiga aytiladi:

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ marta}}$$

a son (takrorlanuvchi ko‘paytuvchi) *darajaning asosi, n* son (ko‘paytuvchi necha marta takrorlanishini ko‘rsatuvchi son) *daraja ko‘rsatkichi* deyiladi.

Masalan,

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81,$$

bu yerda 3 — darajaning asosi, 4 — daraja ko‘rsatkichi, 81 esa 3^4— darajaning qiymati.

Xususan, sonning birinchi darajasi deb, shu sonning o‘ziga aytiladi:

$$a^1 = a.$$

Masalan, $5^1 = 5$, $25^1 = 25$, $\left(\frac{1}{7}\right)^1 = \frac{1}{7}$.

Darajaning asosi istalgan son bo‘lishi mumkinligini aytib o‘tamiz, masalan,

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32; \qquad \left(\frac{2}{5}\right)^3 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{8}{125};$$

$$(-2)^5 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = -32;$$

$$\left(-\frac{2}{3}\right)^4 = \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) = \frac{16}{81};$$

$$0,2^3 = 0,2 \cdot 0,2 \cdot 0,2 = 0,008;$$

$$(-1)^6 = (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) = 1;$$

$$0^3 = 0 \cdot 0 \cdot 0 = 0; \quad 10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10\,000.$$

Darajaga ko‘tarish amali — uchinchi bosqich amal. Agar ifodada qavslar bo‘lmasa, u holda avval uchinchi bosqich amallar, keyin ikkinchi bosqich amallar (ko‘paytirish va bo‘lish), va nihoyat, birinchi bosqich amallar (qo‘shish va ayirish) bajarilishini eslatib o‘tamiz.

Masala. Hisoblang: $7 \cdot 2^4 - 5 \cdot 3^2$.

$$7 \cdot 2^4 - 5 \cdot 3^2 = 7 \cdot 16 - 5 \cdot 9 = 112 - 45 = 67.$$

Sonlarni daraja yordamida yozishdan juda ko‘p hollarda, masalan, natural sonlarni xona qo‘shiluvchilari yig‘indisi shaklida yozish uchun foydalaniladi:

$\triangle$ $3245 = 3 \cdot 1000 + 2 \cdot 100 + 4 \cdot 10 + 5 = 3 \cdot 10^3 + 2 \cdot 10^2 + 4 \cdot 10 + 5$. $\blacktriangle$

Katta sonlarni yozish uchun ko‘pincha 10 sonining darajalari qo‘llaniladi. Masalan, Yerdan Quyoshgacha bo‘lgan masofa taxminan 150 mln km ga teng bo‘lib, uni $1{,}5 \cdot 10^8$ km shaklida yoziladi: Yer sharining radiusi taqriban 6,37 mln. m ga teng, u $6{,}37 \cdot 10^6$ m kabi yoziladi; Yerdan eng yaqin yulduz (Sentavrning α si)gacha bo‘lgan masofani $4 \cdot 10^{13}$ km shaklida yoziladi.

10 dan katta bo‘lgan har bir sonni $a \cdot 10^n$ shaklida yozish mumkin, bunda $1 \le a < 10$ va n — natural son. Bunday yozuv *sonning standart shakli* deyiladi.

Masalan,

$4578 = 4{,}578 \cdot 10^3$, $45{,}78 = 4{,}578 \cdot 10$, $103000 = 1{,}03 \cdot 10^5$.

Fizika va kimyo fanlarini o‘rganishda, mikrokalkulatorda hisoblashlarda va boshqa ko‘p hollarda sonning standart shakldagi yozuvidan foydalaniladi.

Mashqlar

Yig‘indini ko‘paytma shaklida yozing **(121—122):**

121. 1) $4+4+4+4+4$; 3) $c+c+c$;
2) $6+6+6+6$; 4) $a+a+a+a+a$.

122. 1) $2m+2m+2m$;

2) $17ab+17ab+17ab$;

3) $(c-2d)+(c-2d)$;

4) $(3b-a)+(3b-a)+(3b-a)$;

5) $\underbrace{3+3+\ldots+3}_{\text{21 marta}}$;

6) $\underbrace{5+5+\ldots+5}_{\text{17 marta}}$;

7) $\underbrace{m+m+\ldots+m}_{n\text{ marta}}$;

8) $\underbrace{b+b+\ldots+b}_{k\text{ marta}}$.

Ko‘paytmani daraja shaklida yozing **(123—125):**

123. 1) $2\cdot2\cdot2\cdot2\cdot2$; 2) $\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}$; 3) $\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)$;
4) $(-2,7)\cdot(-2,7)\cdot(-2,7)\cdot(-2,7)$.

124. 1) $x\cdot x\cdot x\cdot x\cdot x$;

2) $m\cdot m\cdot m\cdot m\cdot m$;

3) $(2a)\cdot(2a)\cdot(2a)$;

4) $(-3b)\cdot(-3b)\cdot(-3b)\cdot(-3b)$.

125. 1) $(x-y)\cdot(x-y)\cdot(x-y)$;

2) $(a+b)\cdot(a+b)$;

3) $\frac{3x}{2}\cdot\frac{3x}{2}$;

4) $\frac{m}{n}\cdot\frac{m}{n}\cdot\frac{m}{n}\cdot\frac{m}{n}\cdot\frac{m}{n}$.

Ko‘paytmaning daraja shaklidagi yozuvidan foydalanib, ifodani soddalashtiring **(126—128):**

126. 1) $2\cdot2\cdot2\cdot15$;

2) $4\cdot4\cdot4\cdot4\cdot21$;

3) $5\cdot5\cdot8\cdot8\cdot8\cdot2\cdot2$;

4) $6\cdot6\cdot7\cdot7\cdot3\cdot3\cdot3$.

127. 1) $1{,}2\cdot1{,}2\cdot2\cdot2\cdot5\cdot5$;

2) $0{,}5\cdot0{,}5\cdot0{,}5\cdot2\cdot2\cdot4\cdot4$;

3) $0{,}3\cdot0{,}3\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{1}{7}$;

4) $\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot2{,}3\cdot2{,}3$.

128. 1) $9\cdot9\cdot9\cdot a\cdot a\cdot a$;

2) $x\cdot x\cdot x\cdot x\cdot3\cdot3$;

3) $\frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y}(x-y)\cdot(x-y)$;

4) $\frac{a}{b}\cdot\frac{a}{b}\cdot(8a-b)\cdot(8a-b)\cdot(8a-b)$.

Ifodani soddalashtiring **(129—130):**

129. 1) $p\cdot p\cdot p\cdot p+q\cdot q$;

2) $a\cdot a+b\cdot b\cdot b\cdot b$;

3) $a\cdot a+a\cdot a+a\cdot a$;

4) $x\cdot x\cdot x+x\cdot x\cdot x$.

130. 1) $\underbrace{c\cdot c+c\cdot c+\ldots+c\cdot c}_{k\text{ marta}}$; 3) $\underbrace{a\cdot a\cdot\ldots\cdot a}_{n\text{ marta}}+\underbrace{b\cdot b\cdot\ldots\cdot b}_{m\text{ marta}}$;

2) $\underbrace{a\cdot a\cdot a+a\cdot a\cdot a+\ldots+a\cdot a\cdot a}_{n\text{ marta}}$; 4) $\underbrace{5\cdot 5\cdot\ldots\cdot 5}_{k\text{ marta}}+\underbrace{a\cdot a\cdot\ldots\cdot a}_{17\text{ marta}}$.

131. Ifodani o‘qing, darajaning asosini, daraja ko‘rsatkichini ayting:

1) 3^2; 3) $\left(-\frac{2}{9}\right)^{41}$; 5) $(4m+n)^{15}$;

2) $\left(1\frac{3}{8}\right)^3$; 4) $(-1,2)^{39}$; 6) $\left(\frac{2a}{3b}\right)^7$.

Hisoblang **(132—139)**:

132. 1) 2^3; 2) 3^2; 3) 4^4; 4) 5^3.

133. 1) 1^5; 2) $(-1)^7$; 3) 0^{15}; 4) 0^5.

134. 1) $\left(\frac{2}{3}\right)^3$; 2) $\left(\frac{3}{5}\right)^2$; 3) $\left(1\frac{2}{7}\right)^2$; 4) $\left(2\frac{1}{3}\right)^3$.

135. 1) $(2,5)^2$; 2) $(1,7)^2$; 3) $(-0,2)^3$; 4) $(-0,2)^4$.

136. 1) $(-5)^3$; 2) -5^3; 3) $\left(-2\frac{1}{4}\right)^2$; 4) $-\left(2\frac{1}{4}\right)^2$.

137. 1) $\frac{(-0,2)^4}{(0,1)^5}$; 2) $\frac{(0,3)^3}{(-0,1)^4}$; 3) $\frac{(3,2)^2}{(1,6)^2}$; 4) $\frac{(2,6)^2}{(1,3)^2}$.

138. 1) $2\cdot(-3)^2$; 2) $-5\cdot(-2)^3$; 3) $-\frac{1}{2}\cdot(-4)^2$; 4) $-\frac{2}{3}\cdot(-3)^2$.

139. 1) $(-5)^2\cdot\left(-\frac{3}{5}\right)$; 2) $(-3)^3\cdot\left(-\frac{2}{3}\right)$;

3) $-(-3)^2\cdot 2^3$; 4) $-(-3)^2\cdot(-2)^3$.

140. $-x^2;(-x)^2;(-x)^3$ ifodaning qiymatini $x=1\frac{1}{2};\ -5$ da toping.

141. x^2 ifodaning qiymatini x ning jadvalda keltirilgan qiymatlari uchun hisoblang:

x	0	1	–1	2	–2	3	–3	4	–4	5	–5	6	–6
x^2													

142. x^3 ifodaning qiymatini x ning jadvalda ko‘rsatilgan qiymatlari uchun hisoblang:

x	0	1	–1	2	–2	3	–3	4	–4	5	–5	6	–6
x^3													

143. Quyidagi da’volarning qaysi biri to‘g‘ri, qaysi biri noto‘g‘ri? Sababini tushuntiring. Da’vo noto‘g‘ri deb aytsangiz, uni rad etuvchi misol toping.

1) ikkita sonning kvadratlari teng bo‘lsa, bu sonlarning o‘zlari ham teng;

2) ikkita sonning kublari teng bo‘lsa, bu sonlarning o‘zlari ham teng;

3) agar manfiy songa uning kvadrati qo‘shilsa, musbat son hosil bo‘ladi;

4) agar manfiy sondan uning kvadrati ayirilsa, manfiy son hosil bo‘ladi;

5) agar musbat sondan uning kvadrati ayirilsa, musbat son hosil bo‘ladi.

Quyidagi da’volarning qaysi biri to‘g‘ri, qaysi biri no to‘g‘ri? Sababini tushuntiring. Mos misollar tuzing **(144–145):**

144. 1) natural sonning kvadrati ixtiyoriy raqam bilan tugashi mumkin;

2) natural sonning kubi ixtiyoriy raqam bilan tugashi mumkin.

145. 1) natural sonning to‘rtinchi darajasi faqat 0; 1; 5; 6 raqamlaridan biri bilan tugashi mumkin.

2) natural sonning beshinchi darajasi shu son qaysi raqam bilan tugagan bo‘lsa, o‘sha raqam bilan tugaydi.

10- §. Natural ko‘rsatkichli darajaning xossalari

Darajaga ko‘tarish bir nechta muhim xossalarga ega.

1- xossa.

$$a^m \cdot a^n = a^{m+n}.$$

Bir xil asosli darajalarni ko‘paytirishda asos o‘zgarmasdan qoladi, daraja ko‘rsatkichlari esa qo‘shiladi.

○ Natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$2^2 \cdot 2^3 = \underbrace{(2\cdot 2)}_{2\text{ marta}} \cdot \underbrace{(2\cdot 2\cdot 2)}_{3\text{ marta}} =$ | $a^m \cdot a^n = \underbrace{(a\cdot a\cdot a\cdot \ldots \cdot a)}_{m\text{ marta}} \times \underbrace{(a\cdot a\cdot a\cdot \ldots \cdot a)}_{n\text{ marta}} =$

ko‘paytirishning guruhlash qonuniga ko‘ra

$= \underbrace{2\cdot 2\cdot 2\cdot 2\cdot 2}_{5\text{ marta}} =$ | $= \underbrace{a\cdot a\cdot a\cdot \ldots \cdot a}_{(m+n)\text{ marta}} =$

natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$= 2^5.$ | $= a^{m+n}.$

Shunday qilib,

$2^2 \cdot 2^3 = 2^{2+3}.$ | $a^m \cdot a^n = a^{m+n}.$ ●

2- xossa.

$$a^m : a^n = a^{m-n},\ m > n,\ a \neq 0.$$

Bir xil asosli darajalarni bo‘lishda asos o‘zgarmasdan qoladi, daraja ko‘rsatkichlari esa ayiriladi.

○ Shartga ko‘ra

$5 > 3.$ | $m > n,\ a \neq 0.$

Darajaning birinchi xossasiga ko‘ra

$2^{5-3} \cdot 2^3 = 2^5.$ | $a^{m-n} \cdot a^n = a^m.$

Shuning uchun

$2^{5-3} = 2^5 : 2^3.$ | $a^{m-n} = a^m : a^n.$

Shunday qilib,

$2^5 : 2^3 = 2^{5-3}$. | $a^m : a^n = a^{m-n}$, $m > n$, $a \neq 0$. ●

$\frac{a^n}{a^n} = 1$, $a \neq 0$ ekanligini ta'kidlaymiz.

3- xossa.

$$(a^m)^n = a^{mn}.$$

Darajani darajaga ko'tarishda asos o'zgarmasdan qoladi, daraja ko'rsatkichlar esa o'zaro ko'paytiriladi.

○ Natural ko'rsatkichli darajaning ta'rifiga ko'ra

$(2^3)^2 = 2^3 \cdot 2^3 =$ | $(a^m)^n = \underbrace{a^m \cdot a^m \cdot a^m \cdot \ldots \cdot a^m}_{n \text{ marta}} =$

darajaning birinchi xossasiga ko'ra

$= 2^{3+3} =$ | $= \overbrace{a^{m+m+\ldots+m}}^{n \text{ marta}} =$

ko'paytirishning ta'rifiga ko'ra

$= 2^{3 \cdot 2}$. | $= a^{mn}$.

Shunday qilib,

$(2^3)^2 = 2^{3 \cdot 2}$. | $(a^m)^n = a^{mn}$. ●

4- xossa.

$$(ab)^n = a^n b^n.$$

Ko'paytmani darajaga ko'tarishda har bir ko'paytuvchi shu darajaga ko'tariladi.

○ $(2 \cdot 3)^3 = \underbrace{(2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3)}_{3 \text{ marta}} =$ | $(ab)^n = \underbrace{(ab)(ab)\ldots(ab)}_{n \text{ marta}} =$

ko'paytirishning guruhlash va o'rin almashtirish qonuniga ko'ra

$= \underbrace{(2 \cdot 2 \cdot 2)}_{3 \text{ marta}} \cdot \underbrace{(3 \cdot 3 \cdot 3)}_{3 \text{ marta}} =$ | $= \underbrace{(a \cdot a \cdot \ldots \cdot a)}_{n \text{ marta}} \underbrace{(b \cdot b \cdot \ldots \cdot b)}_{n \text{ marta}} =$

natural ko'rsatkichli darajaning ta'rifiga ko'ra

$= 2^3 \cdot 3^3$. | $= a^n \cdot b^n$.

Shunday qilib,

$(2 \cdot 3)^3 = 2^3 \cdot 3^3$. | $(ab)^n = a^n b^n$. ●

5- xossa.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n};\ \ b \neq 0.$$

Kasrni darajaga ko‘tarishda uning surat va maxraji xuddi shu darajaga ko‘tariladi.

○ Natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$$\left(\frac{2}{3}\right)^3 = \underbrace{\left(\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\right)}_{3\text{ marta}} = \quad \left(\frac{a}{b}\right)^n = \underbrace{\left(\frac{a}{b}\cdot\frac{a}{b}\cdots\frac{a}{b}\right)}_{n\text{ marta}} =$$

$$a^m \cdot a^n = a^{m+n}$$
$$a^m : a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$(ab)^n = a^n \cdot b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

kasrlarni ko‘paytirish qoidasiga ko‘ra

$$= \underbrace{\frac{2\cdot 2\cdot 2}{3\cdot 3\cdot 3}}_{3\text{ marta}} = \quad = \underbrace{\frac{a\cdot a\cdots a}{b\cdot b\cdots b}}_{n\text{ marta}} =$$

natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$$= \frac{2^3}{3^3}. \quad = \frac{a^n}{b^n}.$$

Shunday qilib,

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}. \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ b \neq 0. \ ●$$

1- masala. Hisoblang: $\dfrac{11^7 \cdot 7^3 \cdot 3^4}{11^6 \cdot 7 \cdot 3^4}$.

△ $\dfrac{11^7 \cdot 7^3 \cdot 3^4}{11^6 \cdot 7 \cdot 3^4} = 11^{7-6} \cdot 7^{3-1} \cdot 1 = 11 \cdot 49 = 539.$ ▲

2- masala. Yorug‘likning tarqalish tezligi $3 \cdot 10^8$ m/s ga yaqin, Yerdan Quyoshgacha bo‘lgan o‘rtacha masofa $1{,}5 \cdot 10^{11}$ m. Yorug‘lik nuri Quyoshdan Yergacha bo‘lgan masofani qancha vaqtda bosib o‘tadi?

△ Tekis harakatda bosib o‘tilgan yo‘lning $s = vt$ formulasiga asosan:

$$1{,}5 \cdot 10^{11} = 3 \cdot 10^8 \cdot t,$$

bu yerdan $t = \dfrac{1{,}5 \cdot 10^{11}}{3 \cdot 10^8} = 0{,}5 \cdot 10^3 = 500$ (s).

Javob: 500 s = 8 min 20 s. ▲

Ko‘paytmani daraja shaklida yozing **(146—152):**

146. 1) $3^5 \cdot 3^4$; 2) $7^2 \cdot 7^4$; 3) $6^3 \cdot 6$; 4) $5 \cdot 5^5$.

147. 1) $c^3 c^2$; 2) $a^3 a^4$; 3) $\left(\frac{1}{2}a\right)^7 \left(\frac{1}{2}a\right)$; 4) $(3b)(3b)^6$.

148. 1) $(-2)^2 \cdot (-2)^3$; 3) $(-0{,}5)^4 \cdot (-0{,}5)^2$;
2) $(-3)^2 \cdot (-3)^2$; 4) $(-1{,}2)^3 \cdot (-1{,}2)^4$.

149. 1) $2^3 \cdot 2^2 \cdot 2^4$; 3) $(-5)^6 \cdot (-5)^3 \cdot (-5)^4$;
2) $3^2 \cdot 3^5 \cdot 3^3$; 4) $(-6)^3 \cdot (-6)^2 \cdot (-6)^7$.

150. 1) $(1{,}3)^2 \cdot (1{,}3) \cdot (1{,}3)^5$; 3) $y^4 y^3 y^7$;

2) $\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^4$; 4) $b^6 b^8 b$.

151. 1) $(-2{,}5a)^3 (-2{,}5a)^8$; 3) $(x-a)^7 (x-a)^{10}$;

2) $\left(-\frac{5x}{6}\right)^5 \left(-\frac{5x}{6}\right)^7$ 4) $(n+m)^{15}(n+m)^5$.

152. 1) $4^4 \cdot 4^5$; 2) $3^8 \cdot 3^n$;
3) $c^{28} c^n$; 4) $a^n a^{13}$ (n — natural son).

153. Darajani bir xil asosli ikkita darajaning ko‘paytmasi shaklida yozing:

1) 3^4; 2) $\left(\frac{5}{9}\right)^5$; 3) y^3; 4) c^{10}; 5) $(-x)^{17}$; 6) $(-11b)^{43}$.

Sonlarni asosi 2 bo‘lgan daraja shaklida yozing **(154—157):**

154. 1) 32; 2) 4; 3) 2; 4) 128.

155. 1) 16; 2) 64; 3) 256; 4) 1024.

156. 1) $2 \cdot 2^6$; 2) $2^4 \cdot 2^3 \cdot 2^7$; 3) $8 \cdot 2^7$; 4) $16 \cdot 2^5$.

157. 1) $2^7 \cdot 128$; 3) $2^n \cdot 8$;
2) $2^{10} \cdot 32 \cdot 256$; 4) $16 \cdot 2^n$ (n — natural son).

Sonlarni asosi 3 bo‘lgan daraja shaklida yozing **(158—161):**

158. 1) 9; 2) 3; 3) 27; 4) 81.

159. 1) 729; 2) 243; 3) $3 \cdot 3^4$; 4) $3^6 \cdot 3$.

№ 5 *Sonning o‘nli yozuvidagi oxirgi raqam nechaga teng:*
1) 846^{847}; 2) 1987^{1987}; 3) 1998^{1998}; 4) 2009^{2009}?

160. 1) $3^5 \cdot 3^{17} \cdot 3$; 2) $3^2 \cdot 3^{11} \cdot 3^5$; 3) $3^5 \cdot 27$; 4) $81 \cdot 3^2$.

161. 1) $3^n \cdot 3^2$;
2) $3 \cdot 3^n$;
3) $3^{n+1} \cdot 81$;
4) $27 \cdot 3^n$ (n — natural son).

Bo‘linmani daraja shaklida yozing **(162—164):**

162. 1) $7^{10} : 7^8$; 2) $4^3 : 4$; 3) $(0{,}2)^4 : (0{,}2)^3$; 4) $10^{12} : 10^4$.

163. 1) $\left(-\frac{9}{7}\right)^8 : \left(-\frac{9}{7}\right)^5$; 2) $\left(\frac{1}{17}\right)^{18} : \left(\frac{1}{17}\right)^{17}$; 3) $x^{21} : x^7$; 4) $d^{24} : d^{12}$.

164. 1) $\left(\frac{3y}{4}\right)^6 : \left(\frac{3y}{4}\right)^2$;
2) $(2a)^5 : (2a)^3$;
3) $(a - b)^7 : (a - b)^5$;
4) $(m + n)^{10} : (m + n)^5$.

Sonlarni asosi 2 bo‘lgan daraja shaklida yozing **(165—166):**

165. 1) $2^3 : 2$; 2) $2^4 : 4$; 3) $64 : 4$; 4) $32 : 2^3$.

166. 1) $8 : 2^2$; 2) $256 : 32$; 3) $\frac{2^7}{2^5}$; 4) $\frac{2^{10}}{2}$.

Sonlarni asosi 3 bo‘lgan daraja shaklida yozing **(167—168):**

167. 1) $3^5 : 3^2$; 2) $3^4 : 3$; 3) $3^4 : 9$; 4) $27 : 3^2$.

168. 1) $243 : 27$; 2) $81 : 9$; 3) $\frac{3^{15}}{3}$; 4) $\frac{3^8}{3^4}$.

Hisoblang **(169—171):**

169. 1) $\frac{2 \cdot 3^3}{3^2}$; 2) $\frac{2^4 \cdot 3^2}{2^3 \cdot 3}$; 3) $\frac{3^5 \cdot 3^{10}}{3^6 \cdot 3^7}$; 4) $\frac{5^8 \cdot 5^7}{5^4 \cdot 5^9}$.

170. 1) $\frac{8\cdot 3^3}{2\cdot 3^2}$; 2) $\frac{11^3\cdot 4^2}{11^2\cdot 4}$; 3) $\frac{2^4\cdot 2^6\cdot 2^3}{2^5\cdot 2^7}$; 4) $\frac{3^6\cdot 3^3}{3^5\cdot 3\cdot 3}$.

171. 1) $\frac{(-5)^9}{5^7}$; 2) $\frac{6^8}{(-6)^7}$; 3) $\frac{6^6}{3^4\cdot 2^3}$; 4) $\frac{3^6\cdot 2^7}{6^5}$.

Tenglamani yeching **(172—174):**

172. 1) $x : 3^2 = 3^3$; | 2) $x : 2^4 = 2^2$; | 3) $x \cdot 2^6 = 2^8$; | 4) $x \cdot 3^5 = 3^8$.

173. 1) $5^5x = 5^7$; 2) $4^6x = 4^8$; 3) $3^8 : x = 3^8$; 4) $2^{11} : x = 2^9$.

174. 1) $\frac{x}{2^3} = 2^2$; 2) $\frac{x}{3^2} = 3^3$; 3) $\frac{2^8}{x} = 2^5$; 4) $\frac{3^9}{x} = 3^7$.

Ifodani asosi a bo‘lgan daraja shaklida yozing **(175—177):**

175. 1) $(a^5)^6$; 2) $(a^8)^7$; 3) $(a^2)^5a^8$; 4) $a^5(a^2)^8$.

176. 1) $a^7\,a^5\,(a^2)^4$; | 2) $a^3\,(a^3)^3\,a^3$; | 3) $(a^3)^2\,a^4\,(a^4)^3$; | 4) $a^5\,(a^3)^4\,(a^2)^3$.

177. 1) $(a^7)^5 : (a^3)^4$; 2) $(a^6)^4 : (a^3)^5$; 3) $\frac{(a^3)^5a^4}{a^{12}}$; 4) $\frac{a^8(a^4)^4}{(a^3)^4}$.

178. n ning qanday qiymatida tenglik to‘g‘ri bo‘ladi:
1) $3^n = 9$; 2) $128 = 2^n$; 3) $(2^2)^n = 16$; 4) $(3^n)^2 = 81$?

Sonlarni ko‘rsatkichi 2 bo‘lgan daraja shaklida yozing **(179—181):**

179. 1) 0,01; 2) $\frac{25}{36}$; 3) $1\frac{9}{16}$; 4) 0,0004.

180. 1) 5^4; 2) 7^6; 3) $(-0,7)^{14}$; 4) $\left(-\frac{2}{3}\right)^{24}$.

181. 1) a^4; 2) b^6; 3) c^{10}; 4) x^{20}.

Ko‘paytmani darajaga ko‘taring **(182—187):**

182. 1) $(3\cdot 5)^4$; 2) $(7\cdot 6)^5$; 3) $(1,3\cdot 8)^5$; 4) $\left(4\cdot\frac{1}{7}\right)^3$.

183. 1) $(2a)^3$; 2) $(3x)^4$; 3) $(-4x)^5$; 4) $(-8b)^2$.

184. 1) $(ax)^7$; 2) $(6y)^6$; 3) $(2,5cd)^2$; 4) $(3nm)^3$.

185. 1) $(abc)^4$; 2) $(xyz)^7$; 3) $(3\cdot 5\cdot 11)^8$; 4) $(2\cdot 4\cdot 9)^9$.

186. 1) $(xy^3)^2$; 2) $(a^2b)^3$; 3) $(2b^4)^5$; 4) $(0{,}1c^3)^2$.

187. 1) $(10n^2m^3)^3$; 2) $(8a^4b^7)^3$; 3) $(-2{,}3a^3b^4)^2$; 4) $(-2nm^3)^4$.

Ko‘paytmani $3^2b^2=(3b)^2$ namunaga qarab daraja shaklida yozing **(188—190):**

188. 1) 4^5x^5; 2) 2^3a^3; 3) $5^4\cdot 7^4$; 4) $2^5\cdot 3^5$.

189. 1) $\left(\frac{2}{5}\right)^2a^2$; 2) $(3{,}4)^4b^4$; 3) $(-1{,}2)^3y^3$; 4) $\left(-\frac{2}{3}\right)^2a^2$.

190. 1) $16a^2$; 2) $81r^2$; 3) $9^7n^7m^7$; 4) $15^3a^3b^3$.

Ifodani ko‘rsatkichi 2 bo‘lgan daraja shaklida yozing **(191—193):**

191. 1) c^2d^{10}; 2) a^4b^6; 3) $25a^4$; 4) $81m^2$.

192. 1) $a^4b^6c^2$; 2) $x^2y^4z^8$; 3) $49x^8y^6$; 4) $100c^8x^6$.

193. 1) $0{,}25a^{10}b^6$; 2) $0{,}49n^2m^{10}$; 3) $\frac{49}{81}x^{12}y^{14}$; 4) $\frac{16}{625}a^{10}b^{16}$.

Ifodani ko‘rsatkichi 3 bo‘lgan daraja shaklida yozing **(194—197):**

194. 1) a^6; 2) b^9; 3) 5^{15}; 4) 4^6.

195. 1) $(-0{,}2)^{12}$; 2) $\left(-\frac{2}{3}\right)^{15}$; 3) $-0{,}125$; 4) $-0{,}001$.

196. 1) x^3y^9; 2) a^6b^3; 3) $b^9c^{12}d^3$; 4) $x^{12}y^9z^6$.

197. 1) $-27a^3$; 2) $-1000b^6$; 3) $-125n^6m^6$; 4) $-0{,}008x^3y^9$.

Hisoblang **(198—202)**:

198. 1) $(0{,}25)^7\cdot 4^7$; 2) $\left(\frac{4}{5}\right)^{17}\cdot\left(\frac{5}{4}\right)^{17}$;

3) $(-0{,}125)^{11}\cdot 8^{11}$; 4) $(-0{,}2)^5\cdot 5^5$.

199. 1) $(-0{,}25)^9\cdot(-4)^9$; 3) $\left(\frac{6}{11}\right)^3\cdot(8{,}5)^3$;

2) $\left(-\frac{2}{7}\right)^7\cdot(-3{,}5)^7$; 4) $\left(\frac{1}{9}\right)^5\cdot(4{,}5)^5$.

200. 1) $\frac{2^8 \cdot 3^8}{6^5}$; 2) $\frac{4^5 \cdot 3^5}{12^3}$; 3) $\frac{10^5}{2^5 \cdot 5^5}$; 4) $\frac{14^4}{2^3 \cdot 7^3}$.

201. 1) $\frac{6^{12} \cdot 4^{12}}{3^{12} \cdot 8^{12}}$; 2) $\frac{4^{10} \cdot 3^{10}}{2^{10} \cdot 6^{10}}$; 3) $\frac{15^4}{3^4 \cdot 5^2 \cdot 25}$; 4) $\frac{4^{16}}{8^{10}}$.

202. 1) $\frac{8 \cdot 27^3}{3^8}$; 2) $\frac{2^8 \cdot (7^2)^4}{14^7}$; 3) $\frac{16^2 \cdot 3^5}{12^4}$; 4) $\frac{2^9 \cdot (2^2)^5}{(2^5)^3}$.

Kasrni darajaga ko‘taring **(203—206):**

203. 1) $\left(\frac{2}{3}\right)^2$; 2) $\left(\frac{5}{7}\right)^2$; 3) $\left(\frac{3}{a}\right)^2$; 4) $\left(\frac{b}{8}\right)^3$.

204. 1) $\left(-\frac{m}{11}\right)^2$; 2) $\left(-\frac{13}{n}\right)^2$; 3) $\left(\frac{d}{-2}\right)^3$; 4) $\left(\frac{-4}{c}\right)^3$.

205. 1) $\left(\frac{a}{2b}\right)^4$; 2) $\left(\frac{3b}{5c}\right)^4$; 3) $\left(\frac{2^3}{3^2}\right)^7$; 4) $\left(\frac{5^2}{7^4}\right)^3$.

206. 1) $\left(\frac{a+b}{3}\right)^3$; 2) $\left(\frac{7}{2+c}\right)^2$; 3) $\left(\frac{m+n}{m-n}\right)^5$; 4) $\left(\frac{a+b}{a-b}\right)^7$.

Kasrni daraja shaklida yozing **(207—209)**:

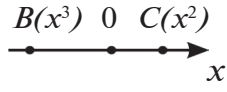
207. 1) $\frac{3^7}{4^7}$; 2) $\frac{2^5}{5^5}$; 3) $\frac{m^3}{2^3}$; 4) $\frac{5^7}{a^7}$.

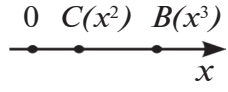
208. 1) $\frac{x^6}{y^6}$; 2) $\frac{a^3}{b^3}$; 3) $\frac{25}{36}$; 4) $\frac{49}{100}$.

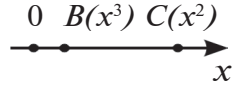
209. 1) $\frac{(2b)^2}{(3b)^2}$; 2) $\frac{(4x)^4}{(3y)^4}$; 3) $\frac{1}{-8}$; 4) $\frac{-1}{27}$.

Hisoblang **(210—211):**

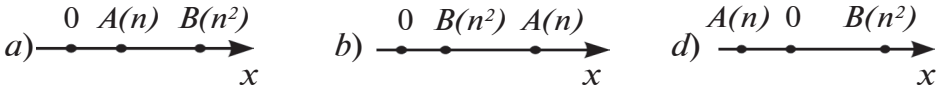
210. $A(x)$ nuqta koordinatalar o‘qining qayerida bo‘lishini chamalab ko‘rsating:

a) 

b) 

d) 

211. $C(n^3)$ nuqta koordinatalar o‘qining qayerida bo‘lishini chamalab ko‘rsating:



212. 1) Yerning massasi $6\cdot10^{24}$ kg ga teng. Quyoshning massasi $2\cdot10^{30}$ kg. Yerning massasi Quyoshning massasidan necha marta kam?

2) Yerdan Sirius deb nomlanuvchi yulduzgacha bo‘lgan masofa 83 000 000 000 000 km. Yorug‘lik nuri Yerdan Siriusgacha necha yilda yetib borishini taqriban hisoblang.

213. Ifodaning son qiymatini toping:

1) $\frac{2-b^2}{2b}$, bunda $b=-2$; 2) $\frac{3a}{a^3-3}$, bunda $a=-3$.

214. Ifodani daraja shaklida yozing:

1) $5^{3n+4}\cdot5^{2n-1}:5^{n+2}$; 3) $\frac{a^{6n-4}a^{4n+1}}{a^{5n-2}}$;

2) $3^{4n+3}\cdot3^{3n-2}:3^{2n-1}$; 4) $\frac{b^{5n-3}b^{3n+2}}{b^{4n-1}}$ (n — natural son).

215. n ning qanday qiymatida tenglik to‘g‘ri bo‘ladi:

1) $(4^4)^n=4^{12}$; 2) $(5^n)^2=5^{14}$; 3) $2^{2n}=4^5$; 4) $3(3^2)^n=3^{11}$?

216. Ko‘paytmani darajaga ko‘taring:

1) $(8a^2b^4c^3)^3$; 2) $(9x^4y^3z^7)^2$;

3) $(-1{,}2x^5y^7z^7)^2$; 4) $(-1{,}2a^3b^2c^4)^5$.

217. Ifodani asosi a bo‘lgan daraja shaklida yozing:

1) $\frac{a^8a^5}{a^3a^6}$; 2) $\frac{a^9a^6}{a^5a^8}$; 3) $\frac{(a^3)^4(a^4)^3}{a^6a^9}$; 4) $\frac{a^6(a^3)^5}{(a^4)^2a^9}$.

218. Sonlardan qaysi biri katta:

1) 54^4 mi yoki 21^{12} mi; 3) 100^{20} mi yoki 9000^{10} mi;

2) 10^{20} mi yoki 20^{10} mi; 4) 6^{20} mi yoki 3^{40} mi?

219. To‘g‘ri tenglik hosil qiling. Masala nechta yechimga ega:

1) $(\ldots)^2\cdot(\ldots)^3=-4a^8b^9c^{11}$; 2) $(\ldots)^2\cdot(\ldots)^3=-8a^{11}b^5c^7$?

220. Tenglamani yeching:

1) $x : 1{,}75 = 7{,}125 - 3\frac{1}{8}$;

2) $\frac{5}{12} + \frac{1}{18} = \frac{17}{12}x$;

3) $18{,}9 : x = 0{,}021 \cdot 100$;

4) $754{,}5 : (37{,}1 + x) = 15$.

221. Sonni standart shaklda yozing:

1) 26 000; 2) 8 647 000; 3) 384 000;

4) Yerdan Quyoshgacha bo‘lgan masofa 149 500 000 km.

11-§ Birhad va uning standart shakli

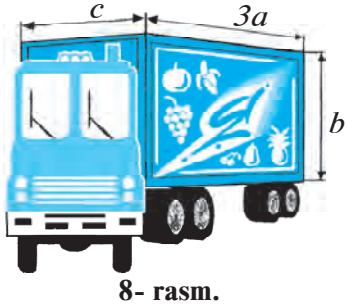
Turli masalalarni yechishda ko‘pincha ab, $\frac{1}{2}abc$, $3a^2b$ ko‘rinishdagi algebraik ifodalarga duch kelinadi. Masalan, o‘lchamlari 8- rasmda ko‘rsatilgan sovitgichli mashina sig‘imi $3abc$ ga teng.

$3abc$ ifoda birinchisi raqam bilan, qolgan uchtasi a, b, c harflari bilan belgilangan to‘rtta ko‘paytuvchining ko‘paytmasidir.

Raqamlar bilan yozilgan ko‘paytuvchilar *sonli ko‘paytuvchilar*, harflar bilan belgilangan ko‘paytuvchilar esa *harfiy ko‘paytuvchilar* deyiladi. Sonli va harfiy ko‘paytuvchilar ko‘paytmasidan iborat algebraik ifoda *birhad* deyiladi.

Masalan, ushbu ifodalar birhadlardir:

$$abc,\ (-4)a \cdot 3ab,\ \frac{1}{4}a(-0{,}3)bab.$$



8- rasm.

Teng ko‘paytuvchilar ko‘paytmasini natural ko‘rsatkichli daraja shaklida yozish mumkin bo‘lganligi uchun sonning darajasi va sonlar darajalarining ko‘paytmasi ham birhadlar deyiladi. Masalan, ushbu ifodalar birhadlar bo‘ladi:

$$\left(\frac{3}{4}\right)^2,\ (-7),\ c^5,\ 4a^2,\ \left(-\frac{1}{2}\right)a^2b.$$

Har bir sonni shu son bilan birning ko‘paytmasi shaklida yozish mumkin bo‘lgani uchun a, 2, $\frac{3}{8}$ ko‘rinishdagi ifodalar ham birhadlar deb hisoblanadi.

Masala. Birhadning qiymatini hisoblang:

$$16ac \cdot (0,5)\, a \cdot (0,25)\, b,$$

bunda $a = \frac{1}{3}$, $b = 34$, $c = \frac{9}{17}$.

△ Harflarning qiymatlarini birhadga qo‘yib, uning qiymatini topamiz, ya’ni yettita sonning ko‘paytmasini hisoblaymiz:

$$16 \cdot \frac{1}{3} \cdot \frac{9}{17} \cdot 0,5 \cdot \frac{1}{3} \cdot 0,25 \cdot 34.$$

Sonlarning birinchisini ikkinchisiga, ular qanday yozilgan bo‘lsa, xuddi shu tartibda ko‘paytirish mumkin:

$$16 \cdot \frac{1}{3} = \frac{16}{3};\ \frac{16}{3} \cdot \frac{9}{17} = \frac{48}{17};\ \frac{48}{17} \cdot 0,5 = \frac{24}{17};$$
$$\frac{24}{17} \cdot \frac{1}{3} = \frac{8}{17};\ \frac{8}{17} \cdot \frac{1}{4} = \frac{2}{17};\ \frac{2}{17} \cdot 34 = 4.$$

Ko‘paytirishning o‘rin almashtirish va guruhlash qonunlarini qo‘llab, hisoblashni qisqacha bajarish ham mumkin:

$$16ac\,(0,5)\,a\,(0,25)\,b = (16 \cdot 0,5 \cdot 0,25)\,(a \cdot a)\,bc = 2a^2bc.$$

Endi $a = \frac{1}{3}$, $b = 34$, $c = \frac{9}{17}$ bo‘lganda $2a^2bc$ birhadning qiymatini topamiz:

$$2 \cdot \left(\frac{1}{3}\right)^2 \cdot 34 \cdot \frac{9}{17} = \frac{2 \cdot 34 \cdot 9}{9 \cdot 17} = 4. \ ▲$$

Masalani ikkinchi usul bilan yechishda berilgan birhad ancha sodda ko‘rinishda yozilgan edi: $2a^2bc$. Bu — birhadning *standart shakliga* misol.

Umuman, birinchi o‘rinda turgan faqat bitta son ko‘paytuvchidan va har xil asosli harfiy darajalardan tuzilgan birhadni *standart shakldagi birhad* deyiladi.

Har qanday birhadni standart shaklda yozish mumkin. Buning uchun barcha son ko‘paytuvchilarni o‘zaro ko‘paytirish va ularning ko‘paytmasini birinchi o‘ringa yozish kerak. So‘ngra bir xil harfiy ko‘paytuvchilar ko‘paytmasini daraja shaklida yozish kerak. Harfiy ko‘paytuvchilar ko‘pincha, shart bo‘lmasa ham, alifbo tartibida joylashtiriladi.

Birhadning standart shaklida bir xil harflar yo‘qligini eslatib o‘tamiz.

Standart shaklda yozilgan birhadning son ko‘paytuvchisini shu *birhadning koeffitsiyenti* deyiladi.

Masalan, $2a$ birhadning koeffitsiyenti 2 ga teng; $\frac{5}{6}ab^2$ birhadning koeffitsiyenti $\frac{5}{6}$ ga teng, $(-7)a^2b^3c$ birhadning koeffitsiyenti (-7) ga teng. Oxirgi holda birhadni qavssiz yozish mumkin:

$$(-7)\,a^2b^3c = -7a^2b^3c\,.$$

1 ga teng bo‘lgan koeffitsiyent, odatda, yozilmaydi, chunki birga ko‘paytirgan bilan son o‘zgarmaydi. Masalan, $1\cdot abc^2 = abc^2$, ya’ni abc^2 birhadning koeffitsiyenti birga teng.

Agar koeffitsiyent (-1) ga teng bo‘lsa, bu holda ham birni va qavslarni yozmasdan, faqat „–“ ishorasini qoldirish mumkin. Masalan, $(-1)abc = -abc$, ya’ni $-abc$ birhadning koeffitsiyenti -1 ga teng.

Mashqlar

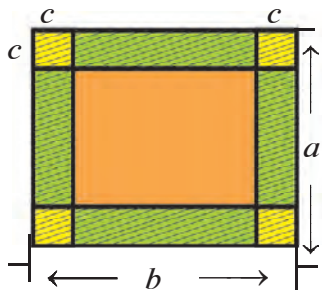
So‘z orqali aytilgan fikrni algebraik ifoda yordamida yozing **(222—224):**

222. 1) a va b sonlar ko‘paytmasining ikkilangani;
2) b va c sonlar ko‘paytmasining uchlangani;
3) x va y sonlar kvadratlarining ko‘paytmasi;
4) a son bilan b son kvadratining ko‘paytmasi.

223. 1) m sonning kubi bilan p sonning ko‘paytmasi;
2) a sonning kvadrati bilan b son ko‘paytmasining uchlangani.

224. 1) t soatdagi sekundlar soni;
2) n metrdagi santimetrlar soni.

225. 1) Berilgan o‘lchamlar bo‘yicha shtrixlangan yuzni hisoblash formulasini chiqaring (9- rasm);
2) $2bc + 2c(a - 2c) = 2ac + 2c(b - 2c)$ tenglikning to‘g‘riligini shakl yordamida ko‘rsating;
3) Shtrixlangan yuzni ikkita to‘g‘ri to‘rtburchak yuzlarining ayirmasi sifatida tasvirlang. Bundan foydalanib, $ab - (b - 2c)(a - 2c) = 2ac + 2c \cdot (b - 2c)$ tenglikni isbotlang.



9- rasm.

226. Birhadning son qiymatini toping:

1) $\frac{3}{4}a^3$, bunda $a = -2$;

2) $0{,}5b^2$, bunda $b = -4$;

3) $3abc$, bunda $a = 2$, $b = \frac{1}{2}$, $c = \frac{1}{3}$;

4) $4pqr$, bunda $p = \frac{1}{2}$, $q = 3$, $r = \frac{1}{6}$;

5) $\frac{1}{7}m^2(-0{,}2)n$, bunda $m = 3$, $n = -35$;

6) $\frac{1}{9}y(-0{,}3)x^2$, bunda $y = -15$, $x = 6$.

227. Birhadni standart shaklda yozing:

1) $3m^2m$;
2) z^5z^5z;
3) $ab\ 0{,}5$;
4) $(-m)(-m^3)$;
5) $5^2pq^2(-4)pq$;
6) $2^3qp^2(-3)^2pq$.

228. Birhadni standart shaklda yozing va son qiymatini toping:

1) $ac12c$, bunda $a = -\frac{1}{3}$, $c = 4$;

2) $\frac{1}{6}a8b^2\ \frac{3}{4}ba^3$, bunda $a = -2$, $b = \frac{1}{2}$;

229. *(Qadimiy masala.)* Hovuzga 4 ta quvur o‘tkazilgan bo‘lib, birinchi quvur hovuzni bir kunda, ikkinchi quvur ikki kunda, uchinchi quvur uch kunda, to‘rtinchi quvur to‘rt kunda to‘ldiradi. To‘rtala quvur birgalikda hovuzni qancha vaqtda to‘ldiradi?

12-§ Birhadlarni ko‘paytirish

Quyidagi masalani yechaylik.

Masala. To‘g‘ri burchakli parallelepipedning hajmi $V = abc$ formula bo‘yicha hisoblanadi, bu yerda a — parallelepipedning bo‘yi, b — eni va c — balandligi. Agar shu parallelepipedning bo‘yi 5 marta, eni $2n$ marta, balandligi $3n$ marta uzaytirilsa, yangi parallelepipedning hajmi qanday bo‘ladi?

△ Yangi parallelepipedning o‘lchamlarini topamiz: bo‘yi $5a$, eni $2nb$, balandligi $3nc$. Bu holda uning hajmi

$$V_1 = (5a)\cdot(2nb)\cdot(3nc)$$

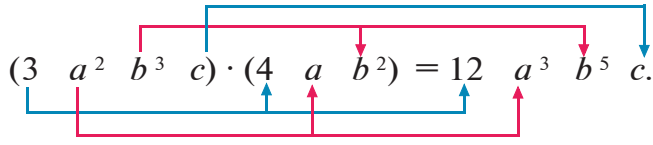
bo‘ladi. ▲

$(5a)\cdot(2nb)\cdot(3nc)$ ifoda quyidagi uchta birhadning ko‘paytmasidir: $5a$, $2nb$, $3nc$. Sonlarni ko‘paytirish qoidalariga ko‘ra bunday tenglikni yozish mumkin:

$$(5a)\cdot(2nb)\cdot(3nc) = 5a\cdot 2nb\cdot 3nc = (5\cdot 2\cdot 3)(annbc) = 30n^2abc.$$

Birhadlarni ko‘paytirish natijasida yana birhad hosil bo‘ladi va uni standart shaklda yozib, soddalashtirish lozim, masalan,

$$(3a^2b^3c)\cdot(4ab^2) = 3a^2b^3c\cdot 4ab^2 = 12a^3b^5c.$$



Ikki yoki bir nechta bir xil birhadlarning ko‘paytmasini, ya’ni birhadning darajasini qaraymiz, masalan, $(5a^3b^2c)^2$. Bu birhad 5, a^3b^2c ko‘paytuvchilarning ko‘paytmasi bo‘lgani uchun ko‘paytmani darajaga ko‘tarish xossasiga ko‘ra:

$$(5a^3b^2c)^2 = 5^2(a^3)^2(b^2)^2c^2 = 25a^6b^4c^2.$$

Xuddi shu kabi:

$$(2pq^2)^3 = 2^3p^3(q^2)^3 = 8p^3q^6.$$

Birhadni natural ko‘rsatkichli darajaga ko‘tarish natijasida yana birhad hosil bo‘ladi.

Birhadlarni ko‘paytiring **(230—237):**

230. 1) $(2a)(3b)$; 2) $(3a)(2b)$; 3) $b^2(-3b^3)$; 4) $(-2a)a^2$;

231. 1) $(2p)(-3c^2)$;
2) $(-5m^2)(-7n)$;
3) $(4a^2)(6a^3)$;
4) $\left(-\frac{1}{2}b^3\right)(8b^2)$.

232. 1) $(0,3a^2)\left(\frac{1}{4}b^3\right)$;
2) $(-8m^3)(0,25n)$;
3) $(0,2p)(-1,3q^2)$;
4) $\left(-\frac{3}{7}c^2\right)\left(-\frac{5}{6}b^3\right)$.

233. 1) $(3ab)(-2a^2b)$;
2) $(-4x^2y)(-7xy^2)$;
3) $(8ab^2)\left(\frac{1}{4}ac^2\right)$;
4) $(6a^2b)\left(\frac{1}{3}bc^2\right)$.

234. 1) $(3a^2b^5c)(6a^3bc^2)$;
2) $(7a^5b^2c)(-3ab^4c)$;
3) $\left(\frac{2}{3}a^2b^3x\right)\left(\frac{3}{4}a^3bx^2\right)$;
4) $\left(-\frac{3}{2}a^3xy^3\right)\left(\frac{3}{4}ay^2\right)$;

235. 1) $(-0,4x^5y^6z^2)(-1,2xyz^3)$;
2) $(-2,5n^4m^5r^2)(3nm^2r^5)$;
3) $\left(-1\frac{1}{3}x^2y^3z\right)\left(-1\frac{1}{2}xy^2z^3\right)$;
4) $\left(2\frac{1}{4}a^2b^5c^3\right)\left(-3\frac{1}{3}a^3b^2c^4\right)$.

236. 1) $\left(-\frac{1}{3}m^2\right)(-24n)(4mn)$;
2) $(-18n)\left(-\frac{1}{6}m^2\right)(-5mn)$;
3) $\left(\frac{1}{3}ay^3\right)\left(\frac{3}{4}x^2y\right)(0,2a^3x)$;
4) $(-13a^2bc)(-5ab^2c)(-0,4abc^3)$.

237. 1) $(-a)(3b)(4a^2b)(5ab^2)$;
2) $(5a)(a^2b^2)(-2b)(-3a)$;
3) $(-1,5ab)\left(\frac{1}{4}bc\right)(2ac)(24ab)$;
4) $(1,2a^2)\left(-\frac{1}{3}ab\right)(-5bc)(2c^2)$.

Birhadni darajaga ko‘taring **(238—241):**

238. 1) $(2a)^3$; 2) $(5b)^2$; 3) $(3b^2)^4$; 4) $(2a^3)^2$.

239. 1) $(-3ab)^4$; 2) $(-4ab)^2$; 3) $(-abc)^5$; 4) $(-2xyz)^3$.

240. 1) $(-2a^2b)^3$; 2) $(-a^2bc)^5$; 3) $(-3x^3y)^2$; 4) $(-2x^2y^3)^4$.

241. 1) $\left(\frac{1}{2}nm^2\right)^3$; | 2) $\left(\frac{1}{3}n^2m^2\right)^4$; | 3) $(-0,1a^3b^3)^3$; | 4) $(0,4a^3b^2)^2$.

Amallarni bajaring **(242—243):**

242. 1) $(-2a)^3\left(-3a\right)$;

2) $(-a)^3\left(2a\right)$;

3) $(-0,2bc^2)^2\left(20cx^2\right)$;

4) $(-0,1ab^2c)^2\left(100by^2\right)$.

243. 1) $\left(-1\frac{3}{5}x^3y^2\right)\left(-\frac{1}{2}c^2x^2\right)^3$;

2) $\left(2\frac{1}{4}x^3y\right)\left(\frac{2}{3}xy^2\right)^2$;

3) $(-3bc^2)^3\left(2ab^2\right)^2$;

4) $(-2a^2b)^2\left(-a^2b^3\right)^3$.

244. Birhadni boshqa birhadning kvadrati shaklida yozing:

1) $9a^2$; 2) $16x^4$; 3) $25a^2b^4$;

4) $81x^6y^2$; 5) $36x^{10}y^4$; 6) $1,21a^8b^4$.

245. Birhadlarni ko‘paytiring va hosil bo‘lgan ifodaning qiymatini toping:

1) $\frac{1}{3}a^2\cdot 3a^2b$, bunda $a=-2$, $b=\frac{5}{7}$;

2) $\frac{2}{5}mn\cdot 10n^2$, bunda $m=0,8$, $n=4$;

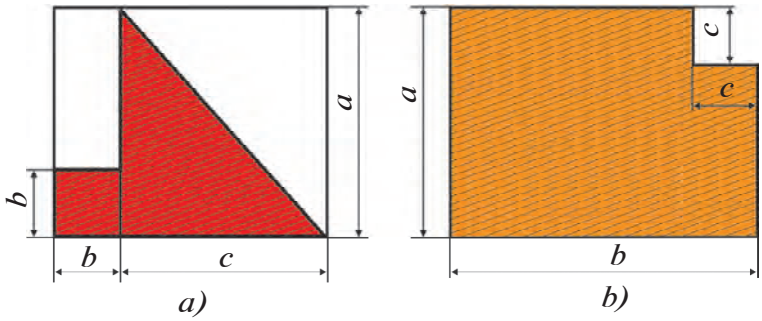
3) $4a\cdot\frac{1}{16}a^2b^2c$, bunda $a=4$, $b=\frac{1}{4}$; $c=3$;

4) $0,7m^2n\cdot 100np$, bunda $m=0,3$, $n=-0,2$, $p=4$.

246. *(Qadimiy masala.)* Baliqning uchdan bir qismi loyda, to‘rtdan bir qismi suv tagida va uch qarichi suv ustida. Baliqning uzunligi necha qarich?

13-§ Ko‘phadlar

Algebrada ko‘pincha birhadlarning yig‘indisi yoki ayirmasidan iborat bo‘lgan algebraik ifodalar qaraladi. Masalan, 10-*a* rasmda tasvirlangan shaklning shtrixlangan qismining yuzi $\frac{1}{2}ac+b^2$ ga teng, 10-*b* rasmda tasvirlangan shaklning yuzi esa $ab-c^2$ ga teng. $\frac{1}{2}ac+b^2$ ifoda ushbu ikkita birhadning yig‘indisi: $\frac{1}{2}ac$ va b^2; $ab-c^2$ ifoda ab va c^2 birhadlarning ayirmasi yoki ab va $(-c^2)$ birhadlarning yig‘indisi. Bu ifodalar birhadlarning algebraik yig‘indisi bo‘ladi. Bunday ifodalar *ko‘phadlar* deyiladi.



10-rasm.

Bir nechta birhadning algebraik yig‘indisi ko‘phad deyiladi. Ko‘phadni tashkil qiluvchi birhadlar shu *ko‘phadning hadlari* deyiladi.

Masalan, $5nm^2-3m^2k-7nk^2+4nm$ ko‘phadning hadlari $5nm^2$, $-3m^2k$, $-7nk$, $4nm$ bo‘ladi.

Ikkita haddan tuzilgan ko‘phad *ikkihad* deyiladi, uchta haddan tuzilgan ko‘phad *uchhad* deyiladi va hokazo.

Ikkihadga misollar: a^2-b^2, $5ab+4c$.

Uchhadga misollar: $a+2b-3c$, $\frac{1}{2}-bc+3ab$.

Birhadni ham ko‘phad deb hisoblaymiz.

Agar ko‘phadning ba’zi hadlari standart shaklda yozilmagan bo‘lsa, u holda shu ko‘phadning barcha hadlarini standart shaklda yozib, uni soddalashtirish mumkin.

Masala. $2a4b-5abac+9bc\frac{1}{3}c$ ko‘phadni soddalashtiring.

△ Berilgan ko‘phadning barcha hadlarini standart shaklda yozamiz:

$$2a4b=8ab;\ -5abac=-5a^2bc;\ 9bc\frac{1}{3}c=3bc^2.$$

Demak, $2a4b-5abac+9bc\frac{1}{3}c=8ab-5a^2bc+3bc^2$. ▲

Mashqlar

247. Ko‘phadni tashkil qiluvchi birhadlarni ayting:

1) $-2x^2+3x-1$;
2) $4x^2-3x+6$;
3) $7a^2-\frac{1}{3}b-\frac{2}{5}c$;
4) $-3a+0,5x-2x^2$.

248. Ko‘phadni birhadlarning yig‘indisi shaklida yozing:

1) $7a^4-9a^3-2a+11$;
2) $-6x^5+3x^4-12x^2+5$;
3) $1,6a^3b-4a^2b^2+13ab^3-b^4$;
4) $2,5x^4-18x^3y-16x^2y-3xy^2$.

249. Birhadlardan ko‘phad tuzing:

1) $6x^2$, $7x$ va 9;
2) $2x^2$, $-11x$ va 3;
3) $-x^4$, x^3 va $-x$;
4) a^5, $-a^4$ va a;
5) $8a$, $4a^2b$, $-2ab^2$ va b^3;
6) $4a^3b$, $-2a^2b^2$, $-5ab^3$.

250. Ko‘phadni, uning har bir hadini standart shaklga keltirib, soddalashtiring:

1) $12a^23ba-2ab3ab^2+11aba$;
2) $2ab^24ab-3a^28aba-2abab^2$;
3) $1,5xy^2(-4)xyz-4mnk5m^2nk$;
4) $4cc^2c\left(-\frac{1}{4}\right)bc+5xy^2xy^2$.

251. Ifodani, uning har bir qo‘shiluvchisini standart shaklga keltirib, soddalashtiring:

1) $3aaa\left(-1\frac{2}{3}ab\right)+4xxx3xy$;

2) $1,5yyy\left(-4xyz\right)-4mnk\cdot 5m^2nk^2$;

3) $\left(2ab\right)\left(\frac{1}{4}a^2b^2\right)-\left(3a^2b\right)\left(\frac{1}{9}b\right)$;

4) $\left(3a\right)\left(\frac{1}{9}ab^2\right)-\left(4b^2\right)\left(\frac{1}{2}a^2b\right)$.

252. Ko‘phadning son qiymatini toping:

1) $2a^3+3ab+b^2$, bunda $a=0,5$, $b=\frac{2}{3}$;

2) $2a^4-ab+2b^2$, bunda $a=-1$, $b=-0,5$;

3) $x^2-2xy+y^2$, bunda $x=y=-4,2$;

4) $x^2+2xy+y^2$, bunda $x=1,2$, $y=-1,2$.

253. Ko‘phadni soddalashtiring va uning son qiymatini toping:

1) $-aba+a^2b2ab^2+4$, bunda $a=2$, $b=\frac{1}{2}$;

2) $b^25ab-5a5a^2b$, bunda $a=\frac{1}{5}$, $b=-2$;

3) $x^2yxy-xy^2xy+xy$, bunda $x=-3$, $y=2$;

4) $xy^2x^2y-xyxy$, bunda $x=-2$, $y=3$.

14-§ O‘xshash hadlarni ixchamlash

Ushbu masalani yechaylik.

1- masala. Har bir sahifasida bir xil sondagi harflar bo‘lgan ikkita kitob bor; har bir sahifadagi satrlar soni n ta va har bir satrdagi harflar soni m ta. Birinchi kitob 300 sahifalik, ikkinchisi 500 sahifalik. Ikkala kitobda hammasi bo‘lib nechta harf bor?

1- usul. Har bir sahifadagi harflar soni mn ta. Birinchi kitobda $300\ nm$ ta harf, ikkinchisida $500\ nm$ ta harf, ikkalasida esa

$$300\ nm + 500\ nm = 800\ nm$$

ta harf bor.

2- usul. Har bir sahifadagi harflar soni mn ga teng. Ikkala kitobdagi sahifalar soni $300 + 500 = 800$ ga, ulardagi harflar soni 800 nm ga teng.

Ikkala javob ham to‘g‘riligi ko‘rinib turibdi, shuning uchun

$$300\ nm + 500\ nm = 800\ nm.$$

Ammo hisoblashlarda ikkinchi usul ancha qulay bo‘ladi. Masalan, agar $n = 40$, $m = 50$ bo‘lsa, u holda $nm = 2\,000$ va 300 nm + 500 nm ifodani hisoblash uchun yana uchta hisoblashni bajarish kerak:

$$300 \cdot 2000 + 500 \cdot 2000 = 600\,000 + 1\,000\,000 = 1\,600\,000.$$

800 nm ifodani hisoblash uchun esa bor-yo‘g‘i bitta amalni bajarish kerak, xolos: $800 \cdot 2000 = 1\,600\,000$.

Mana shuning uchun ham algebraik ifodalarni soddalashtirishni bilish muhim ahamiyatga ega.

300 nm+500 nm ikkihad ikkita birhadning yig‘indisidan iborat:

$$300\ nm \text{ va } 500\ nm.$$

Bu birhadlar bir-biridan faqat koeffitsiyentlari bilan farq qiladi. Bunday birhadlarni *o‘xshash birhadlar* deyiladi. Masalan, abc va $3abc$ birhadlar o‘xshash, $2pq^2$ va $5q^2p$ birhadlar ham o‘xshash, lekin a^2b va ab^2 birhadlar o‘xshash emas.

Bir xil birhadlarni ham o‘xshash deb hisoblaymiz. Masalan, $2a^2b$ va $2a^2b$ birhadlar o‘xshash.

2- masala. $3ab - 2bc + 4ac - ab + 3bc + 4ab$ ko‘phadni soddalashtiring.

△ O‘xshash birhadlarni ajratamiz: $3ab$, $-ab$, $4ab$ birhadlar o‘xshash, ularning tagiga bittadan chiziq chizamiz, $-2bc$ va $3bc$ o‘xshash birhadlarning tagiga ikkitadan chiziq chizamiz. $4ac$ birhadga o‘xshash had yo‘q, uning tagiga chizmaymiz, ya’ni

$$\underline{3ab} - \underline{\underline{2bc}} + 4ac - \underline{ab} + \underline{\underline{3bc}} + \underline{4ab}\ .$$

Ko‘phad hadlarining o‘rinlarini o‘xshash hadlar yonma-yon turadigan qilib almashtiramiz va o‘xshash hadlarni qavs ichiga olamiz:

$$(3ab - ab + 4ab) + (-2bc + 3bc) + 4ac.$$

Ammo

$$3ab - ab + 4ab = (3 - 1 + 4)ab = 6ab,$$
$$-2bc + 3bc = (-2 + 3)bc = bc$$

bo‘lgani uchun

$$3ab - 2bc + 4ac - ab + 3bc + 4ab = 6ab + bc + 4ac. \blacktriangle$$

Ko‘phadlarni o‘xshash birhadlar algebraik yig‘indisi bitta birhad bilan almashtiriladigan bunday soddalashtirish *o‘xshash hadlarni ixchamlash* deyiladi.

$6ab + bc + 4ac$ ko‘phadda har bir had standart shaklda yozilgan va ular orasida o‘xshash hadlar yo‘q. Ko‘phadning bunday shakli *standart shakl* deyiladi.

Har qanday ko‘phadni standart shaklda yozish mumkin. Buning uchun avval ko‘phadning har bir hadini standart shaklda yozish va so‘ngra o‘xshash hadlarni ixchamlash kerak.

3- masala. Ko‘phadni standart shaklga keltiring:

$$6ab\frac{1}{3}ac - 3aca - 8a^2\frac{1}{2}b + 25a^2\frac{1}{5}c + aba - a^2bc.$$

$$\triangle\ 6ab\frac{1}{3}ac - 3aca - 8a^2\frac{1}{2}b + 25a^2\frac{1}{5}c + aba - a^2bc =$$
$$= \underline{2a^2bc} - \underline{\underline{3a^2c}} - \underline{\underline{4a^2b}} + \underline{\underline{5a^2c}} + \underline{\underline{a^2b}} - \underline{a^2bc} =$$
$$= (2a^2bc - a^2bc) + (-3a^2c + 5a^2c) + (-4a^2b + a^2b) =$$
$$= a^2bc + 2a^2c - 3a^2b. \blacktriangle$$

Mashqlar

O‘xshash hadlarni ixchamlang **(254—255):**

254. 1) $\frac{1}{3}x + \frac{1}{2}x + \frac{1}{6}x$;
2) $\frac{5}{6}y - \frac{1}{3}y - \frac{1}{6}y$;
3) $\frac{3}{2}y^4 - \frac{1}{16}y^4 + \frac{1}{32}y^4 - \frac{1}{4}y^4$;
4) $\frac{3}{2}a^2b - \frac{5}{8}a^2b + \frac{1}{8}a^2b - \frac{3}{16}a^2b$.

255. 1) $2m + q + q - 4m$;
2) $3a + 2b - b - a$;
3) $x^2 + 3y^2 + 4x^2 - y^2$;
4) $5a^2 - 4b^2 - 3a^2 + b^2$.

Ko‘phadni standart shaklga keltiring **(256—261):**

256. 1) $11x^2+4x-x^2-4x$; 3) $0,3c^2-0,1c^2-0,5c^3$;

2) $2y^2-3y+2y-2y^2$; 4) $1,2a^2+3,4a^2-0,8a^2$.

257. 1) $\frac{1}{3}x^2-\frac{1}{3}y+\frac{2}{3}x^2+\frac{1}{3}y$; 2) $\frac{1}{5}a^2+\frac{3}{4}b^2+\frac{4}{5}a^2-\frac{3}{4}b^2$;

3) $2ab+0,7b^2-5ab+1,2b^2+8ab$;

4) $5xy-3,5y^2-2xy+1,3y^2-xy$.

258. 1) $-\frac{3}{4}xy+\frac{2}{3}x^2y+xy-\frac{5}{6}x^2y-\frac{1}{2}xy$;

2) $\frac{1}{2}ab^2-\frac{7}{8}ab^2+\frac{3}{4}a^2b-\frac{3}{8}a^2b-\frac{1}{2}ab^2$;

3) $-9,387a-3,89b+8,197a-1,11b-0,81a$;

4) $8,53x-4,73y-5,12x+2,27y+0,59x$.

259. 1) $2a^2b-8b^2+5a^2b+5c^2-3b^2+4c^2$;

2) $8xy^2+4x^3-5x^2y-3x^3+4x^2y-9xy^2$;

3) $\frac{1}{7}ab+\frac{3}{8}a^2-\frac{2}{5}b^3+\frac{6}{7}ab-\frac{3}{8}a^2+\frac{3}{5}b^3$;

4) $\frac{3}{5}ab^2-\frac{2}{3}ab+\frac{1}{4}a^3+\frac{8}{3}ab+\frac{2}{5}ab^2-\frac{3}{4}a^3+\frac{1}{2}a^3$.

260. 1) $5b3b-4c3b-5b2c-4c(-2)c$;

2) $b8b-3c8b+5cb-3c5c$;

3) $6a^22a^2+5b^22a^2-6a^24b^2-5b^24b^2$;

4) $2x^2\frac{1}{2}y-\frac{1}{3}ab3a+1\frac{1}{4}y\frac{4}{5}x^2+aab$.

261. 1) $-9a^2\frac{1}{3}b+a^2b+24a^2\frac{1}{4}c$;

2) $2ab\frac{1}{3}ac-4aca-a^2bc$;

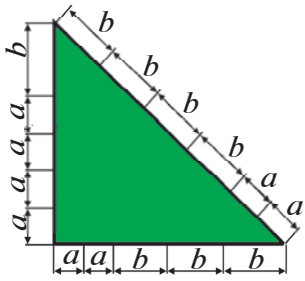
3) $4x^2\frac{1}{2}y-\frac{1}{3}ab9a+4y\frac{4}{5}x^2+aba$;

4) $5a\frac{1}{2}b+\frac{2}{3}a\left(\frac{1}{4}b^2\right)-5b(0,5a)-\frac{1}{3}a^2\left(\frac{1}{15}ab\right)$.

15-§. Ko‘phadlarni qo‘shish va ayirish

O‘lchamlari 11- rasmda ko‘rsatilgan uchburchakni qaraymiz. Uning P perimetri tomonlar uzunliklarining yig‘indisiga teng:

$$P = (2a + 3b) + (4a + b) + (2a + 4b).$$



11- rasm.

Bu ifoda quyidagi uchta ko‘phadning yig‘indisidir:

$$2a + 3b, \quad 4a + b, \quad 2a + 4b.$$

Qavslarni ochish qoidasiga ko‘ra bunday yozish mumkin:

$$P = 2a + 3b + 4a + b + 2a + 4b.$$

O‘xshash hadlarni ixchamlasak,

$$P = 8a + 8b$$

tenglik hosil bo‘ladi.

Ko‘phadlarning istalgan algebraik yig‘indisi ham xuddi shunga o‘xshash soddalashtiriladi, masalan,

$$\left(2n^2 - m^2\right) - \left(n^2 - m^2 + 3q^2\right) = 2n^2 - m^2 - n^2 + m^2 - 3q^2 = n^2 - 3q^2;$$

$$\left(3ab - 4bc\right) + \left(bc - ab\right) - \left(ac - 3bc\right) =$$

$$= 3ab - 4bc + bc - ab - ac + 3bc = 2ab - ac.$$

Bir nechta ko‘phadlarni qo‘shish va ayirish natijasida yana ko‘phad hosil bo‘ladi.

Bir nechta ko‘phadning algebraik yig‘indisini standart shakldagi ko‘phad ko‘rinishida yozish uchun qavslarni ochish va o‘xshash hadlarni ixchamlash kerak.

Ba’zi ko‘phadlarning yig‘indisi yoki ayirmasini sonlarni qo‘shish va ayirishga o‘xshash „ustun“ usulida topish qulay bo‘ladi. Bunda o‘xshash hadlar birining ostiga ikkinchisi turadigan qilib yoziladi, masalan,

1) $\begin{array}{r} 5a-4bc+3ac \\ + \quad 3bc-7ac \\ \hline 5a-bc-4ac \end{array}$;

2) $\begin{array}{r} 5abc-2ab+4ac-bc \\ - \quad 3abc-3ab-ac+3bc \\ \hline 2abc+ab+5ac-4bc \end{array}$.

Mashqlar

Ko‘phadlarning algebraik yig‘indisini toping **(262—267):**

262. 1) $8a+(-3b+5a)$;
2) $5x-(2x-3y)$;
3) $(6a-2b)-(5a+3b)$;
4) $(4x+2)+(-x-1)$.

263. 1) $3x^2-(4x^2+2y)$;
2) $2a^2-(b^2-3a^2)$;
3) $0,6a^2-(0,5a^2-0,4a)$;
4) $1\frac{1}{2}b^2-\left(2b^2-1\frac{1}{4}\right)$.

264. 1) $\left(2\frac{3}{5}b-\frac{3}{4}b^2\right)+\left(\frac{1}{4}b^2-1\frac{3}{5}b\right)$;
2) $(0,1c-0,4c^2)-(0,1c-0,5c^2)$;
3) $(13x-11y+10z)-(-15x+10y-15z)$;
4) $(17a+12b-14c)-(11a-10b-14c)$.

265. 1) $(7m^2-4mn-n^2)-(2m^2-mn+n^2)$;
2) $(5a^2-11ab+8b^2)+(-2b^2-7a^2+5ab)$;
3) $(11ac+13bc+17b^2)-(10ac+10bc-3b^2)$;
4) $(41z+13az+26az^2)-(16z+13az-4az^2)$.

266. 1) $\left(\frac{1}{2}a+\frac{1}{3}b\right)-\left(\frac{5}{2}a-\frac{2}{3}b\right)+(a+b)$;
2) $(0,3a-1,2b)+(a-b)-(1,3a-0,2b)$;
3) $(11p^3-2p^2)-(p^3-p^2)+(-5p^2-3p^3)$;
4) $(5x^2+6x^3)+(x^3-x^2)-(-2x^3+4x^2)$.

267. 1) $(-2x^3+xy^2)+(x^2y-1)+(x^2y-xy^2+3x^3)$;
2) $(3x^2+5xy+7x^2y)-(5xy+3x^2)-(7x^2y-3x^2)$;

3) $\left(8a^2-10ab-b^2\right)+\left(-6a^2+2ab-b^2\right)-\left(a^2-8ab+4b^2\right)$;

4) $\left(4a^2-2ab-b^2\right)-\left(-a^2+b^2-2ab\right)+\left(3a^2+b^2-ab\right)$.

268. Ko‘phadlarning yig‘indisi va ayirmasini toping:

1) $0,1x^2+0,02y^2$ va $0,17x^2-0,08y^2$;

2) $0,1x^2-0,02y^2$ va $-0,17x^2+0,08y^2$;

3) $a^3-0,12b^3$ va $0,39a^3-b^3$;

4) $a^3+0,12b^3$ va $-0,39a^3+b^3$.

269. Ko‘phadlarning yig‘indisini „ustun“ usulida toping:

1) $3ab+a^2-2b^2$ va $2a^2-3ab$;

2) $3x^2+2xy-4y^2$ va $4y^2-2xy+3x^2y^2-x^3$.

270. Ko‘phadlarning ayirmasini „ustun“ usulida toping:

1) $3a^2+8a-4$ va $3+8a-5a^2$;

2) b^3-3b^2+4b va $b+2b^2+b^3$.

271. 1) Agar $P=5a^2+b$, $Q=-4a^2-b$ bo‘lsa, $P+Q$ ifoda nimaga teng?

2) Agar $P=2p^2-3q^3$, $Q=2p^2-4q^3$ bo‘lsa, $P-Q$ ifoda nimaga teng?

3) Agar $A=a^2-b^2+ab$, $B=2a^2+3ab-5b^2$, $C=-4a^2+2ab-3b^2$ bo‘lsa, $A+B+C$ ni toping;

4) Agar $A=2a^2-3ab+4b^2$, $B=3a^2+4ab-b^2$, $C=a^2+2ab+3b^2$ bo‘lsa, $A-B+C$ ni toping.

272. Isbotlang:

1) beshta ketma-ket natural sonning yig‘indisi 5 ga bo‘linadi;

2) to‘rtta ketma-ket natural sonning yig‘indisi 4 ga bo‘linmaydi;

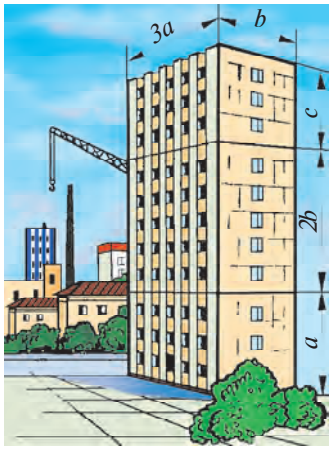
3) to‘rtta ketma-ket toq natural sonning yig‘indisi 8 ga bo‘linadi;

4) to‘rtta ketma-ket juft natural sonning yig‘indisi 4 ga bo‘linadi.

273. Avtobusda n nafar yo‘lovchi bor edi. Dastlabki ikki bekatning har birida m nafardan yo‘lovchi avtobusdan

tushdi, uchinchi bekatda esa hech kim tushmadi, lekin bir necha kishi avtobusga chiqdi, shundan so‘ng avtobusdagi yo‘lovchilar soni k nafar bo‘ldi. Uchinchi bekatda avtobusga necha kishi chiqqan?

16- §. Ko‘phadni birhadga ko‘paytirish



12- rasm.

O‘lchamlari 12- rasmda ko‘rsatilgan to‘g‘ri burchakli parallelepipedni qaraymiz. Uning hajmi asosining yuzi bilan balandligining ko‘paytmasiga teng:

$$(a + 2b + c)(3ab).$$

Bu ifoda $a + 2b + c$ ko‘phad bilan $3ab$ birhadning ko‘paytmasi bo‘ladi.

Ko‘paytirishning taqsimot qonunini qo‘llab, bunday yozish mumkin:

$$(a + 2b + c)(3ab) = a(3ab) + 2b(3ab) + \\ + c\,(3ab) = 3a^2b + 6ab^2 + 3abc.$$

Istalgan ko‘phadni birhadga ko‘paytirish ham xuddi shunday bajariladi, masalan:

$$\left(2n^2m - 3nm^2\right)\left(-4nm\right) = \left(2n^2m\right)\left(-4nm\right) + \left(-3nm^2\right)\left(-4nm\right) = \\ = -8n^3m^2 + 12n^2m^3;$$

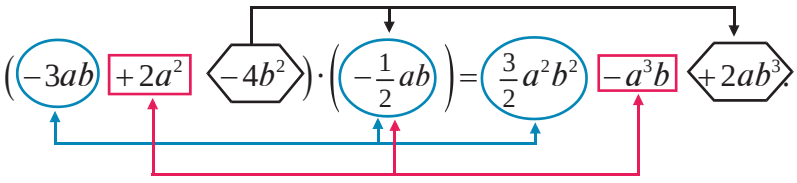
$$\left(3a^2 - 4ab + 5c^2\right)\left(-5bc\right) = 3a^2\left(-5bc\right) - 4ab\left(-5bc\right) + \\ + 5c^2\left(-5bc\right) = -15a^2bc + 20ab^2c - 25bc^3.$$

Ko‘phadni birhadga ko‘paytirish uchun ko‘phadning har bir hadini shu birhadga ko‘paytirish va hosil bo‘lgan ko‘paytmalarni qo‘shish kerak.

Ko‘phadni birhadga ko‘paytirish natijasida yana ko‘phad hosil bo‘ladi. Hosil bo‘lgan ko‘phadni uning barcha hadlarini standart shaklda yozib, soddalashtirish kerak. Oraliqdagi nati-

jalarni yozmasdan, birhadlarni og‘zaki ko‘paytirib, birdaniga javobni yozish ham mumkin, masalan,

$$\left(-3ab+2a^2-4b^2\right)\left(-\frac{1}{2}ab\right)=\frac{3}{2}a^2b^2-a^3b+2ab^3.$$



Birhadni ko‘phadga ko‘paytirish ham shunga o‘xshash bajariladi, chunki ko‘paytuvchilarning o‘rinlarini almashtirish bilan ko‘paytma o‘zgarmaydi, masalan, $4pq\,(3p^2-q+2)=$ $=12p^3q-4pq^2+8pq$.

Mashqlar

Ko‘phad va birhad ko‘paytmasini toping **(274—278):**

274. 1) $(-5)\cdot(10+m)$;
2) $\left(-\frac{1}{2}\right)\cdot(-2+x)$;
3) $(2y-5)\cdot\left(-\frac{1}{7}\right)$;
4) $(-2m+3n)\cdot(-10)$;

275. 1) $(a-b)\,n$;
2) $(-5x+4y)\,2z$;
3) $-6x(5y-2x)$;
4) $\left(x^2-x+1\right)x$.

276. 1) $7ab(2a+3b)$;
2) $5a^2b(15b+3)$;
3) $12p^2q\left(q^2p-q^2\right)$;
4) $3xy^2\left(xy-2x^3\right)$.

277. 1) $17a(5a+6b-3ab)$;
2) $8ab\left(2b-3ac+c^2\right)$;
3) $3x^2y(5x+6y+7z)$;
4) $xyz\left(x^2+2y^2+3z^2\right)$.

278. 1) $\left(\frac{1}{2}a^3b^2-\frac{3}{4}ab^4\right)\frac{4}{3}a^3b$;
2) $\left(\frac{2}{3}a^2b^4+\frac{1}{2}a^3b\right)\frac{3}{2}ab^3$.

Ifodani soddalashtiring **(279—281):**

279. 1) $6(2t-3n)-3(3t-2n)$; 3) $-2(3x-2y)-5(2y-3x)$;
2) $5(a-b)-4(2a-3b)$; 4) $7(4p+3)-6(5+7p)$.

280. 1) $(x^2-1)3x-(x^2-2)2x$;
2) $(4a^2-3b)2b-(3a^2-4b)3b$;
3) $2(3a+4)+3(a-7)-7(2a-7)$;
4) $3(2x-1)-5(x-3)+6(3x-4)$.

281. 1) $5(0,8y-0,1)-0,7(4y+1)+8(0,7-0,4y)$;
2) $3\left(\frac{1}{2}x-1\frac{1}{2}\right)+2\left(\frac{1}{4}x+\frac{1}{2}\right)$; 3) $\frac{5}{4}\left(\frac{1}{5}x-\frac{1}{5}\right)-\frac{4}{5}\left(\frac{1}{4}x-\frac{3}{4}\right)$;
4) $0,2(5y+6)-4(0,25y-1,3)+5(0,1y-1,62)$.

282. Algebraik ifodaning qiymatini toping:
1) $7(4a+3b)-6(5a+7b)$, bunda $a=2, b=-3$;
2) $a(2b+1)-b(2a-1)$, bunda $a=10, b=-5$;
3) $3ab(4a^2-b^2)+4ab(b^2-3a^2)$, bunda $a=10, b=-5$;
4) $4a^2(5a-3b)-5a^2(4a+b)$, bunda $a=-2, b=-3$.

17-§ *Ko‘phadni ko‘phadga ko‘paytirish*

Ushbu masalani qaraylik.

Masala. O‘lchamlari 13- rasmda ko‘rsatilgan shkaflar bilan to‘silgan devor sirtining yuzini toping.

△ Shkaflar bilan band bo‘lgan devorning sirti tomonlari

$$2a+c+2a=4a+c \text{ va } a+b+a=2a+b$$

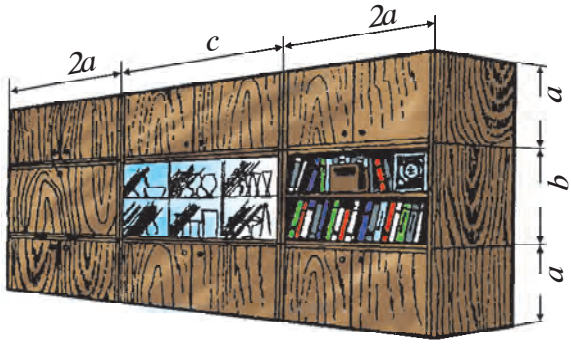
bo‘lgan to‘g‘ri to‘rtburchakdan iborat. Bu to‘g‘ri to‘rtburchakning yuzi $S=(4a+c)(2a+b)$ ga teng. ▲

$(4a+c)(2a+b)$ ifoda $(4a+c)$ va $(2a+b)$ ko‘phadlarning ko‘paytmasidir.

Sonlarni ko‘paytirishning taqsimot qonunini qo‘llab,

$$S = (4a + c)(2a + b) = 4a(2a + b) + c(2a + b)$$

kabi yozish mumkin. So‘ngra, $4a(2a + b) = 8a^2 + 4ab$ va $c(2a + b) = 2ac + bc$ bo‘lgani uchun $S = 8a^2 + 4ab + 2ac + bc$.

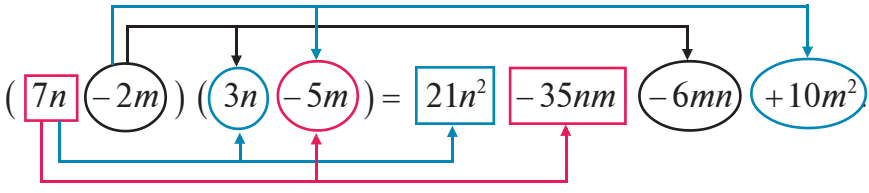


13- rasm.

Shunday qilib, mazkur ko‘phadlarning ko‘paytmasini topish uchun $4a + c$ ko‘phadning har bir hadini $2a + b$ ko‘phadning har bir hadiga ko‘paytirish va natijalarni qo‘shishga to‘g‘ri keldi. Ixtiyoriy ikkita ko‘phadni ko‘paytirish ham xuddi shunday bajariladi, masalan,

$$(7n - 2m)(3n - 5m) = (7n)\cdot(3n) + (7n)\cdot(-5m) + (-2m)\cdot(3n) +$$

$$+(-2m)\cdot(-5m) = 21n^2 - 35nm - 6mn + 10m^2 = 21n^2 - 41nm + 10m^2.$$



Ko‘phadni ko‘phadga ko‘paytirish uchun birinchi ko‘phadning har bir hadini ikkinchi ko‘phadning har bir hadiga ko‘paytirish va hosil bo‘lgan ko‘paytmalarni qo‘shish kerak.

Ko‘phadni ko‘phadga ko‘paytirish natijasida yana ko‘phad hosil bo‘ladi. Hosil qilingan ko‘phadni standart shaklda yozish kerak.

Masalan,

$$(2a-4b+3c)(5b-c)=10ab-2ac-20b^2+4bc+$$

$$+15bc-3c^2=10ab-2ac-20b^2+19bc-3c^2.$$

Bir nechta ko‘phadni ko‘paytirishni navbatma-navbat bajarish kerak, masalan,

$$(a+b)(a+2b)(a-3b)=(a^2+3ab+2b^2)(a-3b)=$$

$$=a^3-3a^2b+3a^2b-9ab^2+2ab^2-6b^3=a^3-7ab^2-6b^3.$$

Mashqlar

Ko‘phadlarni ko‘paytiring **(283—291):**

283. 1) $(a+2)(a+3)$; 3) $(m+6)(n-1)$;
2) $(z-1)(z+4)$; 4) $(b+4)(c+5)$.

284. 1) $(c-4)(d-3)$; 3) $(x+y)(x+1)$;
2) $(a-10)(-a-2)$; 4) $(-p+q)(-1-q)$.

285. 1) $(2x+1)(x+4)$; 3) $(3m-2)(2m-1)$;
2) $(2a+3)(5a-4)$; 4) $(5p-3q)(4p-q)$.

286. 1) $\left(\frac{1}{2}a+3b\right)\left(\frac{1}{2}a-3b\right)$; 3) $\left(\frac{1}{3}a-2b\right)\left(\frac{1}{3}a+2b\right)$;
2) $(0,3-m)(m+0,3)$; 4) $(0,2a+0,5x)(0,2a-0,5x)$.

287. 1) $(a^2+b)(a+b^2)$; 3) $(a^2+2b)(2a+b^2)$;
2) $(5x^2-6y^2)(6x^2-5y^2)$; 4) $(x^2+2x+1)(x+3)$.

288. 1) $(2a-b)(4a^2+2ab+b^2)$;
2) $(3a-2b)(9a^2+6ab+4b^2)$;
3) $(5x+3y)(25x^2-15xy+9y^2)$;
4) $(3a+2b)(9a^2-6ab+4b^2)$.

289. Nuqtalar o‘rniga qanday birhadlarni yozsangiz tenglik to‘g‘ri bo‘ladi:

1) $(2a-5b)(\ldots-\ldots)=6a^3-15a^2b-14ab+\ldots$;

2) $(\ldots-\ldots)(6x^2-5y^2)=12x^3+42x^2y-\ldots-35y^3$;

3) $(3a+4c)(\ldots+\ldots)=20ac+8bc+6ab+\ldots$;

4) $(\ldots+\ldots)(2a+5b)=\ldots+5ab+8ac+20b$?

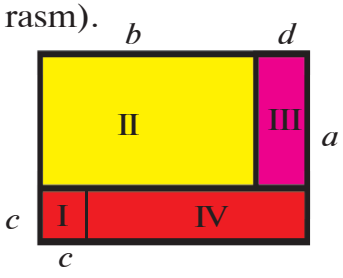
290. 1) $(0,2x+0,2y-z)(x-y)$; 2) $(0,3x-0,3y+z)(x+y)$;

291. 1) $(a-b)(a+b)(a-3b)$; 3) $(x+3)(2x-1)(3x+2)$;

2) $(a+b)(a-b)(a+3b)$; 4) $(x-2)(3x+1)(4x-3)$.

292. 1) Tenglikning to‘g‘riligini isbotlang:
$c^2+b(a-c)+(b+d-c)c+d(a-c)=a(b+d)$;

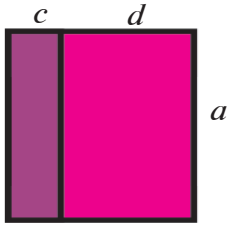
2) To‘g‘ri to‘rtburchakning yuzini hisoblash uchun ikkita ifoda tuzing (14- rasm).

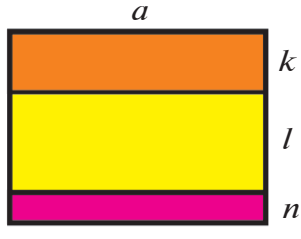


14- rasm.

To‘g‘ri to‘rtburchakning yuzi I, II, III, IV to‘g‘ri to‘rtburchaklar yuzlari yig‘indisiga tengligidan foydalaning va 1- tenglikka geometrik talqin bering.

293. 1) Quyidagi shaklning yuzini va perimetrini hisoblash uchun formulalar tuzing (15- rasm):





15- rasm.

2) Shakl yordamida:

a) $a(c+d)=ac+ad$;

b) $a \cdot (k + l + n) = ak + al + an$ tengliklarni isbotlang. Bu formulalarning geometrik ma'nosini oching.

294. 1) $ABCD$ to'g'ri to'rtburchakning (16- rasm) yuzi

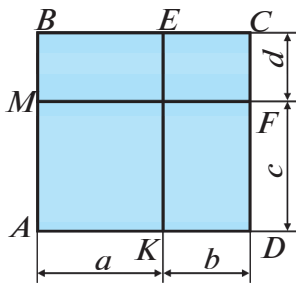
$$(a+b)(c+d) = ac + bc + ad + bd$$

ekanligini ko'rsating.

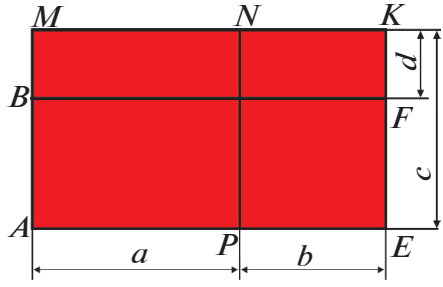
2) $ABFE$ to'g'ri to'rtburchakning (17- rasm) yuzi

$$(a+b)(c-d) = ac + bc - ad - bd$$

ekanligini ko'rsating.



16- rasm.



17- rasm.

18-§ Birhad va ko'phadni birhadga bo'lish

Bir nechta birhad va ko'phadlarni qo'shish, ayirish, ko'paytirish va natural ko'rsatkichli darajaga ko'tarish natijasida yana ko'phad hosil bo'lishi oldingi paragraflarda ko'rsatildi. Sanab o'tilgan bu amallar ichida bo'lish amali uchramadi. Bo'lish amalini o'z ichiga olgan ifodalar V bobda batafsil qaraladi. Ba'zan bo'lish natijasida ham ko'phad hosil bo'ladi.

1. Birhadni birhadga bo'lish

Masala. $32a^3b^2$ birhadni $4a^2$ birhadga bo'ling.

△Sonni sonlar ko'paytmasiga bo'lish xossasidan foydalanamiz: sonni ko'paytmaga bo'lishda shu sonni ko'paytmaning birinchi ko'paytuvchisiga bo'lish kerak, so'ngra hosil bo'lgan natijani ikkinchi ko'paytuvchiga bo'lish kerak va hokazo. Natijada,

$$\left(32a^3b^2\right) : \left(4a^2\right) = \left(\left(32a^3b^2\right) : 4\right) : a^2.$$

Endi ushbu qoidani qo‘llaymiz: *ko‘paytmani songa bo‘lishda ko‘paytmaning ko‘paytuvchilaridan birini shu songa bo‘lish kerak.* U holda

$$\left(32a^3b^2\right) : 4 = \left(32 : 4\right) a^3b^2 = 8a^3b^2;$$
$$\left(8a^3b^2\right) : a^2 = \left(8a^3 : a^2\right) b^2 = 8ab^2.$$

Shunday qilib,

$$\left(32a^3b^2\right) : \left(4a^2\right) = 8ab^2. \blacktriangle$$

Birhadlar boshqa hollarda ham xuddi shunday bo‘linadi, masalan,

$$4a^2b^3 : \left(4a^2b^3\right) = 1;$$
$$\left(66a^4b^2c\right) : \left(22a^2b\right) = 3a^2bc;$$
$$\left(9k^2n^2m^2\right) : \left(-3kn^2m^2\right) = -3k.$$

Bo‘lish natijasini ko‘paytirish bilan tekshirish mumkin: *bo‘linuvchi bo‘luvchining bo‘linmaga ko‘paytmasiga teng bo‘lishi kerak.*

Masalan, $\left(56a^5b^3c\right) : \left(7a^2b^2c\right) = 8a^3b$ — bo‘lish to‘g‘ri bajarilgan, chunki $56a^5b^3c = \left(7a^2b^2c\right) 8a^3b.$

2. Ko‘phadni birhadga bo‘lish

Masala. $2a^2b + 4ab^2 + 8abc$ ko‘phadni $2ab$ birhadga bo‘ling.

△ Ushbu qoidadan foydalanamiz: *yig‘indini songa bo‘lishda har bir qo‘shiluvchini shu songa bo‘lish kerak,* ya’ni

$$\left(2a^2b + 4ab^2 + 8abc\right) : \left(2ab\right) = \left(2a^2b\right) : \left(2ab\right) +$$
$$+ \left(4ab^2\right) : \left(2ab\right) + \left(8abc\right) : \left(2ab\right) = a + 2b + 4c. \blacktriangle$$

Ko‘phadni birhadga boshqa hollarda ham xuddi shunday bo‘linadi, masalan,

$$\left(9a^3b^2 - 3a^2b^3 + a^2b^2\right) : \left(3a^2b^2\right) =$$
$$= \left(9a^3b^2\right) : \left(3a^2b^2\right) + \left(-3a^2b^3\right) : \left(3a^2b^2\right) + \left(a^2b^2\right) : \left(3a^2b^2\right) = 3a - b + \frac{1}{3}.$$

Ko‘phadni birhadga bo‘lish uchun ko‘phadning har bir hadini shu birhadga bo‘lish va hosil bo‘lgan natijalarni qo‘shish kerak.

Ko‘phadni birhadga bo‘lish natijasini ko‘paytirish bilan tekshirish mumkin. Masalan,

$$\left(36n^4m^2 - 45n^2m^4\right) : \left(9n^2m^2\right) = 4n^2 - 5m^2$$

bo‘lish to‘g‘ri bajarilgan, chunki

$$36n^4m^2 - 45n^2m^4 = \left(4n^2 - 5m^2\right)\left(9n^2m^2\right).$$

Ko‘rilgan misollarda birhad (ko‘phad)ni birhadga bo‘lish natijasida birhad (ko‘phad) hosil bo‘ladi. Bunday hollarda ko‘phad birhadga qoldiqsiz bo‘linadi, deyiladi. Ammo ko‘phadni birhadga qoldiqsiz (butun) bo‘lish hamma vaqt ham mumkin bo‘lavermaydi. Masalan, $ab + ac$ ko‘phad ab birhadga qoldiqsiz (butun) bo‘linmaydi.

Birhad (ko‘phad)ni birhadga bo‘lishda harflar bo‘luvchi nolga teng bo‘lmaydigan qiymatlarni qabul qiladi, deb faraz qilinadi.

Mashqlar

Bo‘lishni bajaring **(295–305):**

295. 1) $b^5 : b^2$; 2) $y^{11} : y^7$; 3) $a^7 : a^7$; 4) $b^9 : b^9$.

296. 1) $12x : 4$; 2) $(-15a) : 5$; 3) $(-18y) : 6$; 4) $10c : (-2)$.

297. 1) $8c : (-2)$; 2) $\frac{2}{3}a : 5$; 3) $\left(-\frac{1}{2}b\right) : 2$; 4) $3c : \left(-\frac{1}{3}\right)$.

298. 1) $\frac{2}{5}x : (-2)$; 2) $(-7m) : \left(-\frac{7}{9}\right)$;

3) $\left(-\frac{3}{4}a\right) : \left(-\frac{8}{9}\right)$; 4) $\frac{16}{25} : \left(\frac{4}{5}\right)$.

299. 1) $5a : a$; 2) $8x : x$; 3) $5a : (-a)$; 4) $(-7y) : (-y)$.

300. 1) $(-6x) : (2x)$; 3) $(-6xy) : (-3xy)$;
2) $15z : (5z)$; 4) $12ab : (-4ab)$.

301. 1) $3a : \left(\frac{1}{2}a\right)$; 3) $(-5c) : \left(\frac{1}{3}c\right)$;
2) $\frac{2}{3}b : \left(-\frac{2}{5}b\right)$; 4) $(-1{,}69n) : (1{,}3n)$.

302. 1) $8abc : (-4a)$; 3) $(-6{,}4xy) : (-4x)$;
2) $(-10pq) : (6q)$; 4) $(-0{,}24abc) : (-0{,}6ab)$.

303. 1) $14a^5 : (7a^2)$; 3) $(-0{,}2a^{10}) : (-a^{10})$;
2) $(-42m^7) : (6m)$; 4) $\left(-2\frac{1}{3}a^{17}\right) : (-2a^{17})$.

304. 1) $\left(\frac{1}{3}m^3n^2p^2\right) : \left(-\frac{2}{3}m^2n^2p^2\right)$; 3) $(28{,}9p^2q^2y^3) : (-1{,}7p^2y^3)$;
2) $\left(-1\frac{1}{2}a^4b^3c^2\right) : \left(-\frac{2}{3}a^3bc^2\right)$; 4) $(-6a^3b^2c) : (-2a^2bc)$.

305. 1) $20m^4n^3 : (-5m^2n^3)$; 3) $\left(-\frac{2}{5}a^4x^3y^2\right) : \left(-\frac{1}{2}a^3xy^2\right)$;
2) $(-1{,}3a^3x^2y^3) : (16{,}9a^2xy)$; 4) $\left(-\frac{3}{4}a^5b^3c\right) : \left(-1\frac{1}{2}a^2b^2c\right)$.

306. Ifodani soddalashtiring:
1) $(4a^3b^2)^3 : (2a^2b)^2$; 3) $(-abc^2)^5 : (-a^2bc^3)^2$;
2) $(9x^2y)^3 : (3xy)^2$; 4) $(-x^2y^3z)^4 : (xyz)$.

Bo‘lishni bajaring **(307—310):**

307. 1) $(12a+6) : 3$; 3) $(14m-8) : (-2)$;
2) $(10b-5) : 5$; 4) $(-6+3x) : (-3)$.

308. 1) $(5mn-6np) : n$; 3) $(x-xy) : x$;
2) $(4a^2-3ab) : a$; 4) $(cd-d) : (-d)$.

309. 1) $\left(3a^2b-4ab^3\right):(5ab)$;
2) $\left(2c^5b^4+3c^4b^3\right):\left(-3c^4b^3\right)$;
3) $\left(-27k^4l^5+21k^3l^2\right):\left(-10k^3l^2\right)$;
4) $\left(-a^5b^3+3a^6b^2\right):\left(4a^4b^2\right)$.

310. 1) $\left(6a-8b+10\right):2$;
2) $\left(8x+12y-16\right):(-4)$;
3) $\left(10a^2-12ab+8a\right):(2a)$;
4) $\left(2ab+6a^2b^2-4b\right):(2b)$.

311. Ifodani soddalashtiring:

1) $\left(6a^3-3a^2\right):a^2+\left(12a^2+9a\right):(3a)$;

2) $\left(8x^3-4x^2\right):\left(2x^2\right)-\left(4x^2-3x\right):x$;

3) $\left(3x^3-2x^2y\right):x^2-\left(2xy^2+x^2y\right):\left(\frac{1}{3}xy\right)$;

4) $\left(a^2b-3ab^2\right):\left(\frac{1}{2}ab\right)+\left(6b^3-5ab^2\right):b^2$.

312. Dala hovli to‘g‘ri to‘rtburchak shaklida bo‘lib, uning bo‘yi enidan 1,5 marta uzun. Kanal qazish zarurati bo‘lgani uchun hovlining bo‘yini 6 m ga kamaytirishdi, enini esa 6 m ga uzaytirishdi. Natijada, dala hovlining yuzi avvalgi yuziga qaraganda 84 m^2 ga ortdi. Dala hovlining avvalgi perimetri va yuzini toping.

O‘zingizni tekshirib ko‘ring!

1. Ifodani daraja ko‘rinishida tasvirlang:

$5^3\cdot5^2$; $3^8:3^6$; $(2^3)^4$; $3^5\cdot2^5$.

2. Ifodani soddalashtiring: $(3b+c^2-d)-(c^2-2d)$.

3. Amallarni bajaring:

$(-0,25a^3b^2c)\cdot(5abc)$; $(7m^2-20mn-10m):(10m)$.

4. Ifodani soddalashtiring va uning $m=-0,25$ bo‘lgandagi son qiymatini toping:

$2m(m-1)+(m-2)(m+2)+2m$.

313. Jumlalarni matematik tilda yozing:
1) m sonning kvadratini;
2) a sonning kubini;
3) c va 3 sonlar yig‘indisining kvadratini;
4) c va 3 sonlar kvadratlarining yig‘indisini.

314. Jumlalarni matematik tilda yozing:
1) n va m sonlar ayirmasining kvadratini;
2) n va m sonlar kvadratlarining ayirmasini;
3) n va m sonlar ayirmasining kubini;
4) $\frac{1}{2}$ va b sonlar kublarining ayirmasini.

315. Kvadratning tomoni c metrga teng. Uning perimetri va yuzini yozing.

316. To‘g‘ri to‘rtburchak shaklidagi oynaning bo‘yi enidan 30 sm uzun. Uni deraza romiga solish uchun bo‘yi va enidan 10 sm dan kesishdi. Oynaning kesib tashlangan bo‘laklarining yuzi 1400 sm^2 ga teng. Oynaning dastlabki o‘lchamlarini toping.

317. Bir tomoni ikkinchi tomonidan 3 marta katta bo‘lgan to‘g‘ri to‘rtburchakning bir tomonini x bilan belgilab, uning yuzi formulasini yozing.

318. Qirrasi 1 m bo‘lgan kub qirrasi 1 sm bo‘lgan kublarga ajratilsa va ular ustma-ust qo‘yilsa, qanday balandlikdagi ustun hosil bo‘ladi?

319. Agar odamning yuragi 1 minutda o‘rtacha 75 marta ursa, uning yuragi bir sutka davomida necha marta uradi?

320. O‘quvchi 1 m^3 po‘kakni ko‘tara oladimi? (1 sm^3 po‘kakning massasi 0,2 g).

321. Quyidagi sonlarni standart shaklda yozing:
1) 0° C va 760 mm sim. ust. bosimli 1 sm^3 gazdagi molekulalar soni 27 000 000 000 000 000 000 ga teng;

2) parsek (astronomiyada qabul qilingan uzunlik birligi) 30 800 000 000 000 km ga teng;
3) elektron hisoblash mashinasi 1 sekundda 1 000 000 ta amal bajarishi mumkin.

322. Yer shari sirti 510 mln km^2 dan ortiq. Yer hajmi 1000 mlrd km^3 dan ortiq. Bu sonlarni standart shaklda yozing.

323. 1 *l* dengiz suvida o‘rtacha 0,00001 mg oltin bor. 1 km^3 dengiz suvida qancha oltin bor?

324. Ko‘phadni standart shaklga keltiring:

1) $(2m)(4n)-3a(2b)-(0,2n)(5m)+b(5a)-5nm+8ab$;

2) $13ab-0,2xy-(2a)(5b)+(6x)(0,2y)+a(-3)b$;

3) $2abc5a+1\frac{5}{7}a^2\frac{7}{12}bc-2\frac{2}{3}ab\left(-\frac{3}{8}\right)a$;

4) $3nmk4n-\frac{3}{8}nm\ 2\frac{2}{3}nk+\frac{2}{9}n^2m\left(-4\frac{1}{2}\right)k$.

325. Ko‘phadning qiymatini toping:

1) $-0,08x+73xy^2+27xy^2$, bunda $x=4, y=0,2$;

2) $-2a^2b+4b+11a^2b$, bunda $a=-\frac{1}{3}, b=2\frac{3}{4}$;

3) $5p^3-3p^2+11p-7p-6p^2-7p^2+p$, bunda $p=-1$;

4) $8x^2-7x^3+6x-5x^2+2x^3+3x^2-8x$, bunda $x=1$.

326. Ko‘phadlarning algebraik yig‘indisini toping:

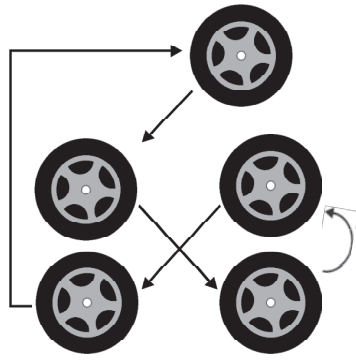
1) $\left(-2x^3+xy^2\right)+\left(x^2y-1\right)+\left(x^2y-xy^2+3x^3\right)$;

2) $\left(3x^2+5xy+7x^2y\right)-\left(5xy+3x^2\right)-\left(7x^2y-3x^2\right)$;

3) $\left(8a^2-10ab-b^2\right)+\left(-6a^2+2ab-b^2\right)-\left(a^2-8ab+4b^2\right)$;

4) $\left(4a^2-2ab+b^2\right)-\left(-a^2+b^2-2ab\right)+\left(3a^2+b^2-ab\right)$.

№ 6 *Yangi „Spark“ avtomobilining egasi yurib turgan va zaxiradagi g‘ildiraklarni rasmda ko‘rsatilgan tartibda almashtirib turdi. 30 000 km yo‘l yurilgach, hamma g‘ildiraklar bir xil yedirilgani ma’lum bo‘ldi. Har bir g‘ildirak necha kilometr yo‘l bosgan (18- rasm)?*



18- rasm.

Ko‘phadlarni ko‘paytiring **(327—328):**

327. 1) $(0,3x+0,3y-z)(x-z)$;

2) $(0,5x-0,5y+z)(x+y)$;

3) $\left(\frac{1}{4}m-\frac{1}{4}n+\frac{1}{5}p\right)(20m+8)$;

4) $\left(0,2a^2-0,4a+1\right)\left(5a^2-10\right)$;

328. 1) $(a-b)(a+b)(2a-3b)$;

2) $(a+b)(a-b)(2a+3b)$;

3) $(x+2)(3x+1)(2x-1)$;

4) $(x-3)(2x+1)(3x-1)$.

329. Bo‘lishni bajaring:

1) $\left(0,01a^4-0,2a^3+0,04a^2+0,002a\right):(0,01a)$;

2) $\left(-0,05x^5-0,08x^4-0,09x^3+0,01x^2\right):\left(-0,01x^2\right)$;

3) $\left(-4m^5n^2-\frac{4}{9}m^4n^5+\frac{2}{3}m^3n^6\right):\left(\frac{2}{3}m^3n^2\right)$;

4) $\left(\frac{3}{4}a^6x^3+\frac{6}{5}a^3x^4-\frac{9}{10}ax^5\right):\left(\frac{3}{5}ax^3\right)$.

III bobga doir sinov mashqlari — testlar

1. Hisoblang: $\left(3^3\cdot 9^5\right):81^3$.

A) 3; B) $\frac{1}{3}$; C) $\frac{1}{9}$; D) $\frac{1}{27}$.

2. Hisoblang: $\dfrac{a^8(b^4)^4}{(b^2)^6\cdot(a^2)^3\cdot(ab)^2}$.

A) a^2b^2; B) b^2; C) a^2; D) $\dfrac{1}{b^2}$.

3. Birhadning son qiymatini toping:

$\dfrac{1}{5}a^2b^3c$, bunda $a=-2,\ b=-1,\ c=10$.

A) $-\dfrac{4}{5}$; B) $\dfrac{4}{5}$; C) -8; D) 8.

4. Birhadni standart shaklda yozing: $2^4ab^2\left(-\dfrac{1}{2}\right)^3a^2b$.

A) $-2a^3b^3$; B) $\dfrac{4}{3}a^3b^3$; C) $-\dfrac{4}{3}b^3a^3$; D) $4a^3b^3$.

5. Birhadlarni ko‘paytiring: $\left(-\dfrac{7}{15}a^3b^2c^3\right)\left(\dfrac{9}{14}ab^2c\right)$.

A) $0,3a^3b^4c^4$; B) $-0,3(abc)^4$;

C) $-\dfrac{9}{15}a^4b^2c^3b^2$; D) $\dfrac{9}{15}a^4c^4b^3$.

6. Ko‘phadni uning har bir hadini standart shaklga keltirib, soddalashtiring: $3b^2a5ab-6b^24aba+ab4ab^2$.

A) $43\ a^3b^3$; B) $43a^2b^3$; C) $-5a^3b^2$; D) $-5a^2b^3$.

7. Ko‘phadlarning algebraik yig‘indisini toping:

$\left(0,5a+\dfrac{2}{3}b\right)-\left(\dfrac{7}{2}a-\dfrac{1}{3}b\right)+2(a+b)$.

A) $a+3b$; B) $-a+3b$; C) $-a-3b$; D) $a-3b$.

8. Ko‘phadni birhadga ko‘paytiring: $\left(4a-\dfrac{1}{3}x\right)\cdot(-3x)$.

A) $-12ax-3x^2$; B) $3x^2-12ax$; C) $3x^2+12ax$;

D) x^2-12ax.

9. Soddalashtiring: $5a(0,4a-b)-4a\left(\dfrac{1}{4}a-b\right)$.

A) $a(a-b)$; B) $a(a+b)$; C) a^2+9ab; D) $3a^2+9ab$.

10. Ko‘phadlarni ko‘paytiring: $(a-b)(a+b)(a^2+b^2)$.

A) a^3-b^4; B) a^4+b^3; C) a^3-b^3; D) a^4-b^4.

11. Bo‘lishni bajaring: $(16a^3b^2-4a^2b^3+a^2b^2):(4a^2b^2)$.

A) $4a-b+\frac{1}{4}$; B) $4a+b+4$;

C) $4ab-\frac{1}{6}+4$; D) $4a-4b+4$.

12. Ifodani soddalashtiring: $\left(18a^4+21a^2\right):\left(3a^2\right)-5a\left(2a+\frac{1}{a}\right)$.

A) $4a^2+2$; B) $16a^2+12$; C) $-4a^2+2$; D) $16a^2+2$.

13. Ko‘phadlarni ko‘paytiring: $(a+2b)(a-2b)(a^2+4b^2)$.

A) a^4-16b^4; B) a^4-8b^3; C) a^3-8b^3; D) a^4+16b^4.

Hisoblang: **(14—16):**

14. $\left(-0,2\right)^5:\left(-0,1\right)^4$.

A) $-3,2$; B) $3,2$; C) $0,00032$; D) $-0,00032$.

15. $-(-3)^3\cdot\left(-\frac{1}{3}\right)^2$.

A) -3; B) 3; C) $-2,7$; D) $\frac{1}{9}$.

16. $\left(5,2\right)^3:\left(1,3\right)^2$.

A) 832; B) $8,32$; C) $83,2$; D) $5,2$.

17. Ko‘phadni birhadga ko‘paytiring:

$\left(\frac{18}{35}a^2-\frac{2}{7}ab+0,6b^2\right)\cdot(-35ab)$.

A) $-18a^3b+10a^2b^2-21ab^3$; B) $-18a^3b-10a^2b^2+21ab^3$;

C) $35a^3b-10ab-28ab^3$; D) $-18a^3-10ab+21a^2b^3$.

18. Hisoblang: $\dfrac{(1,3)^6}{(1,69)^4}\cdot\dfrac{(5,2)^8}{(2,6)^6\cdot 2^{10}}$.

A) 4; B) 2,6; C) 1; D) 1,69.

Tarixiy ma'lumotlar

Noma'lum kattaliklarni harflar bilan belgilash mashhur yunon matematigi Diofant (III asr) asarlaridayoq uchraydi. Koeffitsiyentlarni ham, ma'lum miqdorlarni ham harflar bilan belgilashni F. Viyet (1540—1603) birinchilardan bo'lib qo'llagan. Algebraik tenglamalarni umumiy holda tadqiq qilish harfiy koeffitsiyentlar kiritilgandan keyingina mumkin bo'ldi. F. Viyet undosh bosh lotin harflari — *B, G, D,* ... bilan koeffitsiyentlarni, unli harflari — *A, E, I,* ... bilan esa noma'lumlarni belgilagan. Mashhur fransuz matematigi va faylasufi R. Dekart (1596—1650) koeffitsiyentlarni belgilash uchun lotin alifbosining dastlabki (kichik) harflari *a, b, c, d,* ... dan, noma'lumlarni belgilash uchun esa alifboning oxirgi harflari *x, y, z* lardan foydalangan. Darajaning hozirgi zamonaviy belgilanishi a^2, a^3, ..., a^n (n — natural son)ni ham Dekart kiritgan (1637- yil).

„Al-jabr val muqobala" asarining „Ko'paytirish haqida bob"ida al-Xorazmiy birhadlarni ko'paytirishga, ikkihadni ikki hadga ko'paytirishga hamda soddalashtirishga doir misollarni qaraydi. Al-Xorazmiy misollaridan ba'zilarini keltiramiz:

1) $(10-x)x$;

2) $(10+x)(10+x)$;

3) $(10-x)(10-x)$;

4) $(10-x)(10+x)$;

5) $\left(10+\dfrac{x}{2}\right)\cdot\left(\dfrac{1}{2}-5x\right)$;

6) $(10+x)(x-10)$;

7) $(100+x^2-20x)-(50+10x-2x^2)$;

8) $(100+x^2-20x)+(50+10x-2x^2)$.

Al-Xorazmiy, Ahmad Farg‘oniy, Beruniy, al-Koshiy asarlarida algebraik simvolika bo‘lmagan. Matematik Abu Hasan Ali ibn Muhammad al-Kalasadi (XV asr) asarida algebraik simvolika elementlarini uchratish mumkin. Al-Kalasadi tenglamalarda noma’lumning birinchi darajasini „shay“ so‘zining birinchi harfi bilan, kvadratini „mol“ so‘zining, kubini „ka’b“ so‘zining birinchi harflari bilan belgilagan. Tenglik „=“ belgisi o‘rniga „adala“ (tenglik) so‘zidagi *a* harfini ishlatgan. Biz o‘rganayotgan „Algebra“ kursining simvolikasi (belgilashlar tizimi) XIV—XVII asrlarda shakllangan.

Al-Xorazmiy tenglamalarini yeching:

1) $110-x+\frac{1}{3}\cdot(20+x)-x=4x$;

2) $300-x+\frac{4}{11}\cdot(100-10-x)-20=2x$;

3) $500-x+100-\frac{x}{5}-\frac{3}{4}x=2\cdot\left(100+x+\frac{3}{4}x\right)$;

4) $300-x-\frac{x}{3}+100-\frac{x}{3}x-\frac{x}{3}=4\cdot\left(x+\frac{x}{3}\right)$.

IV BOB
KO‘PHADNI KO‘PAYTUVCHILARGA AJRATISH

19-§ Umumiy ko‘paytuvchini qavsdan tashqariga chiqarish

1- masala. 1- bog‘ tomoni 427 m bo‘lgan kvadrat shaklida. Unga tutashgan 2- bog‘ to‘g‘ri to‘rtburchak shaklida bo‘lib, uning eni 427 m, bo‘yi esa 573 m. Bog‘larning maydoni birgalikda necha gektarni tashkil etadi (19- rasm)?

△ Agar $a=427$ m, $b=573$ m belgilash kiritsak, izlanayotgan maydon $S=a^2+ab$ (m^2) bo‘ladi.

19- rasm.

Bu ifodaga a va b ning qiymatlarini qo‘yib hisoblash vaqtni oladi. Ammo ikkala bog‘ning birgalikdagi maydoni S ni $a\cdot(a+b)$ ko‘paytma ham ifodalaydi, ya’ni $a^2+ab=a\cdot(a+b)$ (rasmga qarang). a^2+ab ifoda unga teng bo‘lgan $a\cdot(a+b)$ ifodaga almashtirilsa, hisoblash ishi ancha soddalashadi. Chindan ham, $a^2+ab=a\cdot(a+b)=427\cdot(427+573)=427\,000$ (m^2) $=42{,}7$ (ga).

Javob: 42,7 ga.▲

Hisoblashlarni soddalashtirish uchun a^2+ab ko‘phad $a\cdot(a+ab)$ ko‘paytma bilan almashtirildi.

Ko‘phadni ikkita yoki bir nechta ko‘phadlar ko‘paytmasi shaklida ifodalash *ko‘phadni ko‘paytuvchilarga ajratish (yoyish)* deyiladi.

Ko‘phadni ko‘paytuvchilarga ajratish algebraik ifodalar ustida amallar bajarishda ham keng qo‘llaniladi.

2- masala. $ab + ac - ad$ ifodaning $a = 43$, $b = 26$, $c = 17$, $d = 23$ bo‘lganda, son qiymatini toping.

△ Hisoblashlarni quyidagicha olib boramiz:

$43 \cdot 26 + 43 \cdot 17 - 43 \cdot 23 = 43 \cdot (26 + 17 - 23) = 43 \cdot 20 = 860$. ▲

Bu yerda ko‘paytirishning taqsimot qonuni qo‘llanilgan:

$$ab + ac - ad = a(b + c - d).$$

$43 \cdot 26 + 43 \cdot 17 - 43 \cdot 23$ sonli ifodada umumiy ko‘paytuvchi 43 soni bo‘ladi; $ab + ac - ad$ algebraik ifodada esa umumiy ko‘paytuvchi a bo‘ladi.

Agar ko‘phadning barcha (son yoki harfiy) hadlari umumiy ko‘paytuvchiga ega bo‘lsa, u holda shu ko‘paytuvchini qavsdan tashqariga chiqarish mumkin.

Qavs ichida berilgan ko‘phadni shu umumiy ko‘paytuvchiga bo‘lish natijasida hosil qilinadigan ko‘phad qoladi.

3- masala. Ushbu ko‘phadni ko‘paytuvchilarga ajrating:

$$6ab + 3b - 12bc.$$

△ Berilgan ko‘phadning barcha hadlari $3b$ umumiy ko‘paytuvchiga ega, chunki

$$6ab = 3b \cdot 2a,\ 3b = 3b \cdot 1,\ -12bc = 3b \cdot (4c).$$

Demak, $6ab + 3b - 12bc = 3b(2a + 1 - 4c)$. ▲

Ko‘phadning umumiy hadini masala mazmuniga qarab, qavsdan tashqariga „+“ ishorasi bilan ham, „–“ ishorasi bilan ham chiqarish mumkin. Misollar keltiramiz:

1) $ab - b = b(a - 1) = -b(1 - a)$;
2) $4a^2b^3 - 6a^3b^2 = 2a^2b^2\ (2b - 3a)$ yoki
$4a^2b^3 - 6a^3b^2 = -2a^2b^2\ (-2b + 3a) = -2a^2b^2\ (3a - 2b)$.

Ko‘phadni umumiy ko‘paytuvchini qavsdan tashqariga chiqarish yo‘li bilan ko‘paytuvchilarga ajratish uchun:

1) *shu umumiy ko‘paytuvchini topish;*
2) *uni qavsdan tashqariga chiqarish kerak.*

Agar ko‘phad hadlarining koeffitsiyentlari natural sonlar bo‘lsa, u holda umumiy ko‘paytuvchini topish uchun ko‘phad

hadlari koeffitsiyentlarining eng katta umumiy bo‘luvchisini topish, bir xil asosli darajalar orasidan esa eng kichik ko‘rsatkichli darajani topish lozimligini ta’kidlab o‘tamiz. Masalan, $28x^2b^3 - 21x^3b^2$ ko‘phadni ko‘paytuvchilarga ajratib, quyidagini hosil qilamiz:

$$7x^2b^2\ (4b - 3x).$$

Bu yerda 7 soni 28 va 21 sonlarining eng katta umumiy bo‘luvchisi, x^2 va b^2 esa x va b ning eng kichik ko‘rsatkichli darajalaridir.

Ko‘phadni ko‘paytuvchilarga ajralganligining to‘g‘riligini hosil bo‘lgan ko‘phadlarni ko‘paytirish yo‘li bilan tekshirish mumkin. Masalan, ko‘paytirishni bajarib, hosil qilamiz:

$$7x^2b^2(4b - 3x) = 28x^2b^3 - 21x^3b^2.$$

Umumiy ko‘paytuvchi ko‘phad bo‘lishi ham mumkin, masalan:

1) $5(a + b) + x(a + b) = (a + b)(5 + x)$;
2) $3x(a - 2b) + 5y(a - 2b) + 2(a - 2b) = (a - 2b)(3x + 5y + 2)$.

Ba’zan umumiy ko‘paytuvchini qavsdan tashqariga chiqarishdan oldin $a - b = -(b - a)$ tenglikni qo‘llash foydali bo‘ladi, masalan:

1) $(a - 3)x - (3 - a)y = (a - 3)x + (a - 3)y = (a - 3)(x + y)$;
2) $15a^2b(x^2 - y) - 20ab^2(x^2 - y) + 25ab(y - x^2) = 15a^2b(x^2 - y) - 20ab^2(x^2 - y) - 25ab(x^2 - y) = 5ab(x^2 - y)(3a - 4b - 5)$.

Mashqlar

330. Sonlarni tub ko‘paytuvchilarga ajrating: 70, 121, 240, 168, 225.

331. Kasrlarni qisqartiring: $\frac{45}{60}$; $\frac{18}{24}$; $\frac{75\cdot15}{25\cdot24}$; $\frac{40\cdot14}{7\cdot15}$.

332. Ko‘paytirishning taqsimot qonunini qo‘llang va hisoblang:

1) $81\cdot17 - 15\cdot81$;
2) $24\cdot2{,}78 + 41\cdot2{,}78$;
3) $15\cdot17 + 15\cdot67$;
4) $14\frac{3}{8}\cdot1\frac{1}{4} - 4\frac{3}{8}\cdot1\frac{1}{4}$.

333. Ko‘paytmani ko‘phad shaklida yozing:

1) $(a+2)(a+3)$; 3) $3c^3(2c^3-5)$;

2) $2x(x-1)$; 4) $(a^2+b)(a-b^2)$.

334. A bekatdan B bekatga tomon motorli qayiq 20 km/soat tezlik bilan jo‘nadi. Oradan ikki soat o‘tgandan keyin A dan B ga tomon ikkinchi motorli qayiq 24 km/soat tezlik bilan yo‘lga chiqdi. Ikkala qayiq ham B ga bir vaqtda yetib keldi. A dan B gacha bo‘lgan masofani toping.

335. 1) 3^6+3^4 ifodaning 30 ga; 90 ga;
2) 7^8+7^6 ifodaning 49 ga; 350 ga;
3) 11^8-11^6 ifodaning 24 ga; 60 ga
karrali ekanini isbotlang.

Umumiy ko‘paytuvchini qavsdan tashqariga chiqaring **(336 — 344)**:

336. 1) $2m+2n$; 2) $3a-3x$; 3) $8-4x$; 4) $6a+12$.

337. 1) $9a+12b+3$; 3) $-10x+15y-5z$;

2) $8a-4b-2$; 4) $9x-3y+12z$.

338. 1) $ax-ay$; 2) $cd+bc$; 3) $xy+2x$; 4) $3x-xy$.

339. 1) $9mn+9n$; 2) $3bd-3ab$; 3) $11z-33yz$; 4) $6pk-3p$.

340. 1) $ab-ac+a^2$; 3) $6a^2-3a+12ba$;

2) $xy-x^2+xz$; 4) $4b^2+8ab-12a^2b$.

341. 1) a^4+2a^2; 3) $a^4b^2+ab^3$;

2) a^4-3a^3; 4) $x^2y^3-x^3y^2$.

342. 1) $18y^7+12y^4$; 3) $15x^5-5x^3$;

2) $6x^4-24x^2$; 4) $6a^5+3a^2$.

343. 1) $9a^2b^2-12ab^3$; 3) $7a^2bc+14ab^2c$;

2) $20x^3y^2+4x^2y$; 4) $9xyz^2-12xy^2z$.

344. 1) $6y^5+12y^4-3y^3$; 3) $4a^2b^2+36a^2b^3+6ab^4$;
2) $20a^4-5a^3+15a^5$; 4) $2x^2y^4-2x^4y^2+6x^3y^3$.

345. Hisoblang:
1) $137^2+137\cdot 63$; 3) $0,7^3+0,7\cdot 9,51$;
2) $187^2-187\cdot 87$; 4) $0,9^3-0,81\cdot 2,9$.

Ko‘paytuvchilarga ajrating **(346—349):**

346. 1) $a(m+n)+b(m+n)$; 3) $a(b-5)-(b-5)$;
2) $b(a+5)-c(a+5)$; 4) $(y-3)+b(y-3)$.

347. 1) $2a(a-b)+3b(a-b)$; 3) $5a(x+y)-4b(x+y)$;
2) $3n(m-3)+5m(m-3)$; 4) $7a(c-d)-2b(c-d)$.

348. 1) $a^2(x-y)+b^2(x-y)$; 3) $a(x^2+y^2)-b(x^2+y^2)$;
2) $a^2(x+y)-b^2(x+y)$; 4) $x(a^2-2b^2)+y(a^2-2b^2)$.

349. 1) $2b(x-1)-3a(x-1)+c(x-1)$;
2) $c(p-q)-a(p-q)+d(p-q)$;
3) $x(a^2+b^2)+y(a^2+b^2)-z(a^2+b^2)$;
4) $m(x^2+1)-n(x^2+1)-l(x^2+1)$.

Ko‘paytuvchilarga ajrating **(350—352):**

350. 1) $c(a-b)+b(b-a)$; 3) $(x-y)+b(y-x)$;
2) $a(b-c)-c(c-b)$; 4) $2b(x-y)-(y-x)$.

351. 1) $7(y-3)-a(3-y)$; 3) $b^2(a-1)-c(1-a)$;
2) $6(a-2)+a(2-a)$; 4) $a^2(m-2)+b(2-m)$.

352. 1) $a(b-c)+b^2(b-c)-7(c-b)$;
2) $x(x-y)+y(y-x)-3(x-y)$;
3) $x(a-2)+y(2-a)+(2-a)$;
4) $a(b-3)+(3-b)-b(3-b)$.

353. Tenglamani yeching:

1) $8-(x-3)(x+3)=10-(x-1)^2$;

2) $(2x+1)^2-(2x-3)^2=4(7x-5)$;

3) $x:15=2\frac{1}{12}:14{,}5$;

4) $\frac{x}{2{,}3}=\frac{2{,}1}{9\frac{6}{7}}$.

354. It tulkining orqasidan quvdi. It sekundiga 8 m, tulki esa 6 m tezlik bilan chopmoqda. Ularning orasidagi masofa dastlab 360 m bo‘lgan, tulkining o‘z uyasiga yetib olishi uchun esa 1 km qolgan edi. Tulki o‘z uyasiga yetib olishga ulguradimi?

20-§ Guruhlash usuli

Guruhlash usuli hamma hadlari uchun umumiy ko‘paytuvchi mavjud bo‘lmagan ko‘phadlarga qo‘llaniladi.

Ba’zan, berilgan ko‘phadning bir nechta hadlarini qavs ichiga olib, umumiy ko‘paytuvchini aniqlash mumkin. Ko‘phadni guruhlash usuli qo‘shish va ko‘paytirishning guruhlash, o‘rin almashtirish va taqsimot qonunlariga asoslangan.

Misollar qaraymiz:

1) $a(b+c)+b+c=a(b+c)+(b+c)=(b+c)(a+1)$;

2) $a(b-c)-b+c=a(b-c)-(b-c)=(b-c)(a-1)$.

Birinchi misolda ko‘phadning oxirgi ikkita hadini „ + “ ishorasi bilan, ikkinchi misolda ko‘phadning oxirgi ikkita hadini „ − “ ishorasi bilan qavs ichiga olish yetarli bo‘ldi.

3) $m(3x-y)+3nx-ny=m(3x-y)+(3nx-ny)=$
$=m(3x-y)+n(3x-y)=(3x-y)(m+n)$;

4) $-mx^2-my^2+n(x^2+y^2)=(-mx^2-my^2)+n(x^2+y^2)=$
$=-m(x^2+y^2)+n(x^2+y^2)=(x^2+y^2)(n-m)$.

Uchinchi va to‘rtinchi misollarda ko‘phadning ikkita hadini qavs ichiga olishdan tashqari hosil qilingan har bir guruhda umumiy ko‘paytuvchi qavsdan tashqariga: birinchi holda „ + “ ishorasi bilan, ikkinchisida esa „ – “ ishorasi bilan chiqarildi.

Ba’zan ko‘phad hadlarini turli usullar bilan guruhlash mumkin. Masalan, $2am + 2an - 3bm - 3bn$ ko‘phadni ko‘paytuvchilarga ikki usul bilan ajratish mumkin:

I usul	II usul
$2am + 2an - 3bm - 3bn =$	$2am + 2an - 3bm - 3bn =$
$= (2am + 2an) - (3bm + 3bn) =$	$= (2am - 3bm) + (2an - 3bn) =$
$= 2a(m + n) - 3b(m + n) =$	$= m(2a - 3b) + n(2a - 3b) =$
$= (m + n)(2a - 3b).$	$= (2a - 3b)(m + n).$

Oltita haddan iborat ko‘phadni ko‘paytuvchilarga ajratishga doir misol qaraymiz:

$$ax + bx - ay - by + az + bz = (ax + bx) - (ay + by) + (az + bz) =$$
$$= x(a + b) - y(a + b) + z(a + b) = (a + b)(x - y + z).$$

Bu yerda ko‘phadlar ikkitadan guruhlarga ajratilgan; ularni uchtadan guruhlash ham mumkin edi:

$$ax + bx - ay - by + az + bz = (ax - ay + az) + (bx - by + bz) =$$
$$= a(x - y + z) + b(x - y + z) = (a + b)(x - y + z).$$

Ko‘phadni guruhlash usuli bilan ko‘paytuvchilarga ajratish uchun:

1) ko‘phadning hadlarini, ular ko‘phad shaklidagi umumiy ko‘paytuvchiga ega bo‘ladigan qilib, guruhlarga birlashtiriladi;

2) bu umumiy ko‘paytuvchini qavsdan tashqariga chiqariladi.

Ko‘paytuvchilarga ajrating **(355—360)**:

355. 1) $a+b+c(a+b)$;
2) $m-n+p(m-n)$;
3) $x+3a(x+y)+y$;
4) $x+2a(x-y)-y$.

356. 1) $(x+y)+(x+y)^{2;}$
2) $(a-b)^2+a-b$;
3) $2m(m-n)+(m-n)^2$;
4) $4q(p-1)+(p-1)^2$.

357. 1) $2m(m-n)+m-n$;
2) $4q(p-1)+p-1$;
3) $2m(m-n)-n+m$;
4) $4q(p-1)+1-p$.

358. 1) $a(x-c)+bc-bx$;
2) $a(b+c)+db+dc$;
3) $3a(2b+c)+8b+4c$;
4) $2x(3x-4y)-6x+8y$.

359. 1) $ac+bc-2ad-2bd$;
2) $ac-3bd+ad-3bc$;
3) $2bx-3ay-6by+ax$;
4) $5ay-3bx+ax-15by$.

360. 1) $xy^2-by^2-ax+ab+y^2-a$;
2) $ax^2-ay-bx^2+cy+by-cx^2$.

361. Hisoblang:
1) $139\cdot15+18\cdot139+15\cdot261+18\cdot261$;
2) $125\cdot48-31\cdot82-31\cdot43+125\cdot83$;
3) $14,7\cdot13-2\cdot14,7+13\cdot5,3-2\cdot5,3$;
4) $3\frac{1}{3}\cdot4\frac{1}{5}+4,2\cdot\frac{2}{3}+3\frac{1}{3}\cdot2\frac{4}{5}+2,8\cdot\frac{2}{3}$.

362. Ifodaning son qiymatini toping:
1) $5a^2-5ax-7a+7x$, bunda $x=-3$, $a=4$;
2) $m^2-mn-3m+3n$, bunda $m=0,5$, $n=0,25$;
3) $a^2+ab-5a-5b$, bunda $a=6,6$, $b=0,4$;
4) $a^2-ab-2a+2b$, bunda $a=\frac{7}{20}$, $b=0,15$.

363. Hisoblang:

1) $287^2 - 287 \cdot 48 + 239 \cdot 713$;

2) $73,4^2 + 73,4 \cdot 17,2 - 90,6 \cdot 63,4$.

364. Tenglamani yeching:

1) $x(x-4) + x - 4 = 0$; 2) $t(t+7) - 4t - 28 = 0$.

№ 7 *Ali bilan Valining massasi birgalikda 5 ta tarvuz massasiga teng. Valining massasi 1 ta qovun massasidan 4 marta ko'p. Vali bilan 2 ta qovunning birgalikdagi massasi 3 ta tarvuz massasiga teng. Alining massasi nechta qovunning massasiga teng?*

21-§ Yig'indining kvadrati. Ayirmaning kvadrati

Ikkita son yig'indisining kvadrati $(a + b)^2$ ni qaraymiz. Ko'phadni ko'phadga ko'paytirish qoidasidan foydalanib, hosil qilamiz:

$$\circ\ (a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2,$$

ya'ni

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2.} \bullet \qquad (1)$$

Ikki son yig'indisining kvadrati — birinchi son kvadrati, qo'shuv birinchi son bilan ikkinchi son ko'paytmasining ikkilangani, qo'shuv ikkinchi son kvadratiga teng.

(1) formulani 20- rasmda tasvirlangan kvadratning yuzini ko'zdan kechirib, osongina hosil qilish mumkinligini aytib o'tamiz.

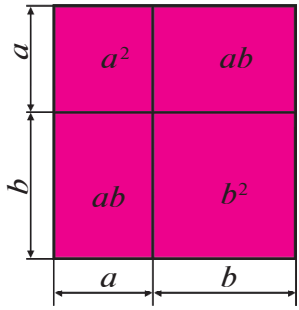
Endi ikki son ayirmasining kvadratini qaraymiz:

$$\circ\ (a-b)^2 = (a-b)(a-b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2,$$

ya'ni

$$\boxed{(a-b)^2 = a^2 - 2ab + b^2.} \bullet \qquad (2)$$

Ikki son ayirmasining kvadrati — birinchi son kvadrati, ayiruv birinchi son bilan ikkinchi son ko‘paytmasining ikkilangani, qo‘shuv ikkinchi son kvadratiga teng.



20- rasm.

(1) va (2) tengliklarda a va b istalgan sonlar yoki algebraik ifodalardir.

(1) va (2) formulalarni qo‘llashga doir misollar:

1) $(2m+3k)^2=(2m)^2+2\cdot 2m\cdot 3k+(3k)^2=4m^2+12mk+9k^2$;

2) $(5a^2-3)^2=(5a^2)^2-2\cdot 5a^2\cdot 3+3^2=25a^4-30a^2+9$;

3) $(-a-3b)^2=((-1)(a+3b))^2=(-1)^2(a+3b)^2=$
$=(a+3b)^2=a^2+2a\cdot 3b+(3b)^2=a^2+6ab+9b^2$.

Zaruriy hisoblashlarni og‘zaki bajarib, oraliq natijalarni yozmaslik mumkin. Masalan, birdaniga bunday yozish mumkin:

$$(5a^2-7b^2)^2=25a^4-70a^2b^2+49b^4.$$

Yig‘indi yoki ayirmaning kvadrati formulalari *qisqa ko‘paytirish formulalari* deyiladi va ba’zi hollarda hisoblashlarni soddalashtirish uchun qo‘llaniladi. Masalan:

1) $99^2=(100-1)^2=10\,000-200+1=9801$;

2) $52^2=(50+2)^2=2500+200+4=2704$.

(1) formula $(1+a)^2$ ifodaning qiymatlarini taqribiy hisoblashlarda ham qo‘llaniladi. a son musbat yoki manfiy son bo‘lib, uning moduli 1ga nisbatan kichik bo‘lsa (masalan, $a=0{,}0032$ yoki $a=-0{,}0021$), u holda a^2 son yanada kichik bo‘ladi va shu sababli

$$(1+a)^2=1+2a+a^2$$

tenglikni $(1+a)^2\approx 1+2a$ taqribiy tenglik bilan almashtirish mumkin. Masalan:

1) $(1{,}002)^2 = (1 + 0{,}002)^2 \approx 1 + 2 \cdot 0{,}002 = 1{,}004$;
2) $(0{,}997)^2 = (1 - 0{,}003)^2 \approx 1 - 2 \cdot 0{,}003 = 0{,}994$.

Yig‘indining kvadrati va ayirmaning kvadrati formulalari ko‘phadni ko‘paytuvchilarga ajratishda ham qo‘llaniladi, masalan:

1) $x^2 + 10x + 25 = x^2 + 2 \cdot 5 \cdot x + 5^2 = (x+5)^2$;

2) $a^4 - 8a^2b^3 + 16b^6 = (a^2)^2 - 2 \cdot a^2 \cdot 4b^3 + (4b^3)^2 = (a^2 - 4b^3)^2$.

Masala. Formulani isbotlang:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3. \qquad (3)$$

○ $(a+b)^3 = (a+b)(a+b)^2 = (a+b)(a^2 + 2ab + b^2) =$

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$. ●

Xuddi shunga o‘xshash,

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \qquad (4)$$

formulani ham isbotlash mumkin.

(3) va (4) formulalar, mos ravishda, *yig‘indining kubi va ayirmaning kubi formulalari* deb ataladi.

(3) va (4) formulalar ham *qisqa ko‘paytirish formulalari* hisoblanadi.

Mashqlar

Quyidagi mashqlarda ikkihadning kvadratini ko‘phad shaklida tasvirlang **(365—372)**:

365. 1) $(c+d)^2$; 3) $(2+x)^2$; 5) $(y+3)^2$;
2) $(x-y)^2$; 4) $(x+1)^2$; 6) $(7+m)^2$.

366. 1) $(m-2)^2$; 3) $(7-m)^2$; 5) $\left(a+\frac{1}{3}\right)^2$;
2) $(x-3)^2$; 4) $(y-6)^2$; 6) $\left(b+\frac{1}{2}\right)^2$.

367. 1) $(q+2p)^2$; 2) $(3x+2y)^2$; 3) $(6a-4b)^2$; 4) $(5z-t)^2$.

368. 1) $(3a^2+1)^2$; 2) $(a^2+1)^2$; 3) $(2x^2+3n^2)^2$; 4) $(x^2+y^2)^2$.

369. 1) $\left(m-\frac{1}{5}\right)^2$; 2) $\left(a-\frac{1}{3}\right)^2$; 3) $\left(\frac{a}{2}-\frac{b}{3}\right)^2$; 4) $\left(\frac{x}{3}+\frac{y}{4}\right)^2$.

370. 1) $(0,2x+0,3y)^2$; 3) $\left(\frac{2}{3}x^3-\frac{3}{4}\right)^2$;

2) $(0,4b-0,5c)^2$; 4) $\left(\frac{1}{4}a^3-\frac{4}{5}\right)^2$.

371. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ formulaga qanday geometrik ma'no bera olasiz?

Nuqtalar o'rniga mos so'zlarni qo'ying:

Qirrasining uzunligi a va b bo'lgan ... yasaymiz. O'lchamlari $a\times a\times b$ va $a\times b\times b$ bo'lgan .... yasaymiz. Ularni shunday taxlaymizki, ... hosil bo'ladi.

372. 1) $(-4ab-5a^2)^2$; 3) $(0,2x^2+5xy)^2$;

2) $(-3b^2-2ab)^2$; 4) $(4xy+0,5y^2)^2$.

Qisqa ko'paytirish formulalaridan foydalanib, amallarni bajaring **(373—375)**:

373. 1) $(90-1)^2$; 2) $(40+1)^2$; 3) 101^2; 4) 98^2.

374. 1) 999^2; 2) 1003^2; 3) 51^2; 4) 39^2.

375. 1) 72^2; 2) 57^2; 3) 997^2; 4) 1001^2.

Ifodani soddalashtiring **(376—377):**

376. 1) $(x-y)^2+(x+y)^2$; 3) $(2a+b)^2-(2a-b)^2$;

2) $(x+y)^2-(x-y)^2$; 4) $(2a+b)^2+(2a-b)^2$.

377. 1) $(a+b)^3+(a-b)^3$; 2) $3(2-a)^2+4(a-5)^2$;

3) $(x-1)^3-(x+1)^3$; 4) $-(3+x)^2+5(1-x)^2$.

Tenglamani yeching **(378—379):**

378. 1) $16x^2-(4x-5)^2=15$; 3) $-5x(x-3)+5(x-1)^2=-20$;

2) $64x^2-(3-8x)^2=87$; 4) $(2x-3)^2-(2x+3)^2=12$.

379. 1) $(3x-1)^2-(3x-2)^2=0$;

2) $(y-2)(y+3)-(y-2)^2=5$;

3) $(x+3)(x+7)-(x+4)^2=0$;

4) $(y+8)^2-(y+9)(y-5)=117$.

380. Ifodaning qiymatini toping:

1) $9a^3-a(3a+2)^2+4a(3a+7)$, bunda $a=-1\frac{1}{6}$;

2) $(2y-5)^2-4(y-3)^2-4y$, bunda $y=-\frac{2}{7}$;

3) $25m(m-1)-(5m-3)^2-6m$, bunda $m=-0,3$;

4) $24x^2-(7x-2)^2+(5x-3)(5x+1)$, bunda $x=-\frac{5}{9}$.

381. x ni shunday birhadga almashtiringki, natijada tenglik bajarilsin:

1) $(x-4b^7)^2=25a^4b^2-40a^2b^8+16b^{14}$;

2) $(x+7c)^2=25b^6+70b^3c+49c^2$;

3) $(2a+x)^3=8a^3+12a^2b+6ab^2+b^3$;

4) $(5b^2-x)^2=25b^4-30a^2b^3+9a^4b^2$.

382. Ifodani ikkihadning kvadrati shaklida tasvirlang:

1) $a^2-10ab+25b^2$; 3) k^4+2k^2+1;

2) $25+10x+x^2$; 4) $p^2-1,6p+0,64$.

383. x ni shunday birhadga almashtiringki, natijada ikkihadning kvadrati hosil bo‘lsin:

1) a^2+4a+x; 3) $36a^2-x+49b^2$;

2) $p^2-0,5p+x$; 4) $a^2-6ab+x$.

384. a ning qanday qiymatlarida ifodani ikkihadning kvadrati ko‘rinishida yozish mumkin:

1) $(3x-5)^2+(4x+12)^2+ax$;

2) $(17x+10)^2-(15x-8)^2+ax$?

385. Isbot qiling:

1) $(a-b)^2=(b-a)^2$; 4) $(a-b)^3=-(b-a)^3$;

2) $(-a-b)^2=(b+a)^2$; 5) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$;

3) $(-a-b)(a+b)=-(a+b)^2$; 6) $(a-b)^3=a^3-3a^2b+3ab^2-b^3$.

22-§ Kvadratlar ayirmasi formulasi

Ikki son yig‘indisini ularning ayirmasiga ko‘paytiramiz:

$$(a+b)(a-b)=a^2-ab+ab-b^2=a^2-b^2,$$

ya’ni

$$(a+b)(a-b)=a^2-b^2. \quad (1)$$

$$a^2-b^2=(a-b)(a+b). \quad (2)$$

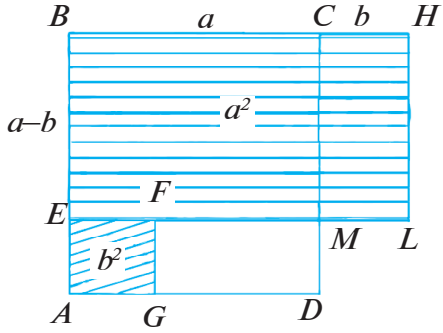
Ikki son kvadratlarining ayirmasi shu sonlar ayirmasi bilan ular yig‘indisining ko‘paytmasiga teng.

(1) va (2) tenglikda a, b istalgan sonlar yoki algebraik ifodalardir, masalan:

1) $(nm+3k)(nm-3k)=n^2m^2-9k^2$;

2) $4a^4b^2-25a^2b^4=(2a^2b+5ab^2)(2a^2b-5ab^2)$;

3) $(a+b)^2-16=(a+b-4)(a+b+4)$.



$S_{ABCD} = a^2;$
$S_{AEFG} = b^2;$
$S_{GFEBCD} = S_{EBHL};$
$S_{GFEBCD} = a^2 - b^2;$
$S_{EBHL} = (a-b)(a+b).$

(2) formulaning geometrik talqini.

(1) formula ham *qisqa ko'paytirish formulasi* deyiladi. Uni hisoblashlarni soddalashtirish uchun qo'llaniladi.

Masalan:

1) $63 \cdot 57 = (60+3)(60-3) = 3\,600 - 9 = 3\,591;$

2) $98 \cdot 102 = (100-2)(100+2) = 100^2 - 2^2 = 10\,000 - 4 = 9\,996.$

(2) tenglik *kvadratlar ayirmasi formulasi* deyiladi. U ko'phadlarni ko'paytuvchilarga ajratishda qo'llaniladi.

Masalan:

1) $a^2 - 9 = a^2 - 3^2 = (a-3)(a+3);$

2) $4b^4 - 0,64c^2 = (2b^2)^2 - (0,8c)^2 = (2b^2 - 0,8c)(2b^2 + 0,8c);$

3) $(a-b)^2 - 1 = (a-b-1)(a-b+1);$

4) $(a+b)^2 - (a-c)^2 = (a+b-a+c)(a+b+a-c) =$
$= (b+c)(2a+b-c).$

Mashqlar

(1) formuladan foydalanib, ko'paytirishni bajaring **(386—394):**

386. 1) $(c+d)(c-d);$ 3) $(a+c)(c-a);$
2) $(p+q)(p-q);$ 4) $(m-n)(m+n);$

387. 1) $(x+5)(x-5)$; 3) $(a-4)(4+a)$;
2) $(a+3)(a-3)$; 4) $(7+x)(x-7)$.

388. 1) $(2b+a)(2b-a)$; 3) $(y+6x)(6x-y)$;
2) $(c+3d)(c-3d)$; 4) $(3m-2n)(2n+3m)$.

389. 1) $\left(4d-\frac{1}{2}\right)\left(\frac{1}{2}+4d\right)$; 3) $\left(\frac{1}{2}y-\frac{1}{3}x\right)\left(\frac{1}{2}y+\frac{1}{3}x\right)$;
2) $\left(\frac{5}{6}a-b\right)\left(b+\frac{5}{6}a\right)$; 4) $\left(\frac{2}{3}m+\frac{3}{4}n\right)\left(\frac{2}{3}m-\frac{3}{4}n\right)$.

390. 1) $(c^2+d^2)(c^2-d^2)$; 3) $(x^4-y^3)(y^3+x^4)$;
2) $(a^2+b^3)(a^2-b^3)$; 4) $(m^3-n^3)(m^3+n^3)$.

391. 1) $(3a^2+4b^3)(3a^2-4b^3)$; 3) $(0,2t^3+0,5p^4)(0,5p^4-0,2t^3)$;
2) $(2m^4-5n^2)(5n^2+2m^4)$; 4) $(1,2a^2-0,3b^2)(1,2a^2+0,3b^2)$.

392. 1) $\left(\frac{3}{4}a^2-\frac{1}{2}b^3\right)\left(\frac{1}{2}b^3+\frac{3}{4}a^2\right)$; 3) $\left(0,5q+\frac{1}{3}p^2\right)\left(0,5q-\frac{1}{3}p^2\right)$;
2) $\left(\frac{2}{3}x^4-\frac{4}{5}y^5\right)\left(\frac{2}{3}x^4+\frac{4}{5}y^5\right)$; 4) $\left(1,5c^2-\frac{3}{4}b\right)\left(\frac{3}{4}b+1,5c^2\right)$.

393. 1) $(3x^2y-4xy^2)(3x^2y+4xy^2)$; 3) $(7ab+x^2y^3)(7ab-x^2y^3)$;
2) $(5ab^2+2a^2b)(5ab^2-2a^2b)$; 4) $(ab^3-4xy)(ab^3+4xy)$.

394. 1) $(3+x)(3-x)(9+x^2)$; 3) $(4x^2+y^2)(2x+y)(2x-y)$;
2) $(x^2+1)(x+1)(x-1)$; 4) $(3a-2b)(3a+2b)(9a^2+4b^2)$.

Qisqa ko‘paytirish formulalaridan foydalanib, hisoblang **(395—396):**

395. 1) $48\cdot52$; 2) $68\cdot72$; 3) $43\cdot37$; 4) $47\cdot53$.

396. 1) $27\cdot33$; 2) $44\cdot36$; 3) $84\cdot76$; 4) $201\cdot199$.

397. Soddalashtiring:

1) $(c-3)^2-(c+3)(3-c)$;

2) $(a+2)^2-(a+2)(2-a)$;

3) $(2x+3y)(2x-3y)+(2x+3y)^2$;

4) $(3a-4b)(3a+4b)-(3a-4b)^2$;

5) $(-b-a)(a+b)+a^2+b^2$;

6) $(b-a)(-a-b)+2b^2$.

398. Ifodaning qiymatini toping:

1) $4m-(m+3)^2+(m-3)(m+3)$, bunda $m=-2,4$;

2) $(3x+4)^2-10x-(x-4)(4+x)$, bunda $x=-0,1$;

3) $2(k-7)(k+5)-(k-5)^2-(k-7)(7+k)$, bunda $k=-\frac{1}{2}$;

4) $(a+3)^2+(a-3)(3+a)-2(a+2)(a-4)$, bunda $a=-\frac{1}{5}$.

399. Tenglamani yeching:

1) $(2x+3)^2-4(x-1)(x+1)=49$;

2) $(3x+4)^2-(3x-1)(1+3x)=49$;

3) $x^3+2x^2-9x-18=0$;

4) $y^3-3y^2-4y+12=0$.

400. Kvadratning ikki qarama-qarshi tomonining har biri 8 sm ga uzaytirildi, qolgan ikki tomoni esa shuncha qisqartirildi. Shaklning yuzi qanday o‘zgardi?

401. Hisoblang: $\frac{5^4\cdot 0,128-5^3\cdot 0,628\cdot 5}{125\cdot 0,25}$.

23-§ Ko‘phadni ko‘paytuvchilarga ajratishning bir necha usulini qo‘llash

Ko‘phadni ko‘paytuvchilarga ajratishda ba’zan bir emas, balki bir necha usullar qo‘llaniladi. Misollar keltiramiz:

1) a^3-a ko‘phadni ko‘paytuvchilarga ajrating:

$$\triangle\ a^3-a=a\left(a^2-1\right)=a\left(a-1\right)\left(a+1\right).\ \blacktriangle$$

Bu yerda ikkita usuldan foydalanilgan: umumiy ko‘paytuvchini qavsdan tashqariga chiqarish va kvadratlar ayirmasi formulasini qo‘llash.

2) $(a^2+1)^2-4a^2$ ko‘phadni ko‘paytuvchilarga ajrating:

$$\triangle\left(a^2+1\right)^2-4a^2=\left(a^2+1\right)^2-\left(2a\right)^2=\left(\left(a^2+1\right)-2a\right)\left(\left(a^2+1\right)+2a\right)=$$
$$=\left(a^2+1-2a\right)\left(a^2+1+2a\right)=\left(a^2-2a+1\right)\left(a^2+2a+1\right)=$$
$$=\left(a-1\right)^2\left(a+1\right)^2.\ \blacktriangle$$

Bu yerda qo‘shiluvchilar umumiy ko‘paytuvchiga ega emasligi sababli, avval kvadratlar ayirmasi formulasidan foydalanildi, so‘ngra yig‘indi va ayirma kvadratlarining formulalaridan foydalanildi. Yana bir misol yechib ko‘raylik:

3) $\triangle\ 4x^2-y^2+4x+2y=\left(4x^2-y^2\right)+\left(4x+2y\right)=$
$$=\left(2x-y\right)\left(2x+y\right)+2\left(2x+y\right)=\left(2x+y\right)\left(2x-y+2\right).\ \blacktriangle$$

Birhadlar umumiy ko‘paytuvchiga ega bo‘lmagani va biror formulani qo‘llash mumkin bo‘lmagani uchun, bu yerda avval guruhlash usulidan foydalanildi, so‘ngra esa kvadratlar ayirmasi formulasi qo‘llanildi.

Ko‘rib chiqilgan bu misollar ko‘phadni ko‘paytuvchilarga ajratishga doir topshiriqlarni bajarishda quyidagi tartibga rioya qilish foydali ekanligini ko‘rsatadi:

1) umumiy ko‘paytuvchini (agar u bor bo‘lsa) qavsdan tashqariga chiqarish;

2) ko‘phadni qisqa ko‘paytirish formulalari bo‘yicha ko‘paytuvchilarga ajratishga urinib ko‘rish;

3) agar oldingi usullar maqsadga olib kelmasa, guruhlash usulini qo‘llashga harakat qilish.

Masala. Tenglikni isbotlang:

$$a^3+b^3=(a+b)\left(a^2-ab+b^2\right). \quad (1)$$

○ Tenglikning o‘ng tomonidagi qavslarni ochamiz:

$$(a+b)\left(a^2-ab+b^2\right)=a^3-a^2b+ab^2+a^2b-ab^2+b^3=a^3+b^3.$$

Tenglikning o‘ng tomoni chap tomoniga tengligi kelib chiqdi, ya’ni (1) tenglik isbot qilindi. ●

Xuddi shu kabi

$$a^3-b^3=(a-b)\left(a^2+ab+b^2\right) \quad (2)$$

tenglikning to‘g‘riligi isbotlanadi.

(1) va (2) tengliklar, mos ravishda, *kublar yig‘indisi va kublar ayirmasi formulalari* deb ataladi. Bu formulalar ham ko‘phadni ko‘paytuvchilarga ajratishda qo‘llaniladi.

Masalan:

1) $27+b^3=3^3+b^3=(3+b)\left(9-3b+b^2\right)$;

2) $x^4-8xy^3=x\left(x^3-8y^3\right)=x\left(x^3-(2y)^3\right)=x(x-2y)\left(x^2+2xy+4y^2\right)$.

Mashqlar

402. Hisoblang:

1) 47^2-37^2; 2) 54^2-44^2;

3) $50,7^2-50,6^2$; 4) $29,4^2-29,3^2$.

403. (Og‘zaki.) Ko‘paytuvchilarga ajrating:

1) $36-x^2$; 2) a^2-25; 3) y^2-1; 4) $1-b^2$.

404. 1) $(a+2b)^2 = a^2+4b^2$; 2) $(2a-3b)^2 = 4a^2-9b^2$ tengliklar haqida nima deya olasiz?
a) ular qaysi a va b larda to‘g‘ri-yu, qaysilarida noto‘g‘ri?
b) ixtiyoriy a va b lar uchun ularni to‘g‘ri bo‘ladigan qilishni uddasidan chiqasizmi?

Ko‘paytuvchilarga ajrating **(405—416):**

405. 1) $25x^2-9$; | 2) $4a^2-9$; | 3) $64y^2-36x^2$; | 4) $81a^2-16b^2$.

406. 1) c^2d^2-9; | 2) a^2b^2-16; | 3) $4a^2-9b^2$; | 4) $16x^2-25y^2$.

407. 1) $\frac{1}{9}y^2-\frac{16}{25}x^2$; 3) $0,25a^2-49b^2$;
2) $\frac{4}{9}a^2-\frac{1}{16}b^2$; 4) $0,09x^2-16y^2$.

408. 1) $36x^2y^2-1$; | 2) x^2y^4-16; | 3) $81a^6-49b^4$; | 4) $25a^2-9b^6$.

409. 1) a^4-b^4; 2) a^4-b^8; 3) a^4-16; 4) b^4-81.

410. 1) $(a+b)^2-c^2$; 3) $(a+2b)^2-9a^2$;
2) $(m-n)^2-k^2$; 4) $(3x-y)^2-4y^2$.

411. 1) $(a+b)^2-(a-c)^2$; 3) $(2a+b)^2-(2b+a)^2$;
2) $(a+b)^2-(b+c)^2$; 4) $(a-3b)^2-(3a+b)^2$.

412. 1) $9a^2-6a+1$; 3) $36b^2+12b+1$;
2) $1+2c+c^2$; 4) $81-18x+x^2$.

413. 1) $9x^2+24x+16$; 3) $36m^2+12mn+n^2$;
2) $100-60a+9a^2$; 4) $a^2+10ab+25b^2$.

414. 1) $x^4+2x^2y+y^2$; 3) $4c^4+12c^2b^3+9b^6$;
2) $p^4-2p^2q+q^2$; 4) $25a^6+30a^3b+9b^2$.

415. 1) a^4-8a^2+16; 3) $25a^4-10a^2b+b^2$;
2) b^4-18b^2+81; 4) $16-8a^2b^2+a^4b^4$.

416. 1) $-a^2-2a-1$; 3) $-2a^2+8ab-8b^2$;
2) $-9+6b-b^2$; 4) $-12ab-3a^2-12b^2$.

417. Ifodaning son qiymatini toping:
1) $5m^2-10mn+5n^2$, bunda $m=142, n=42$;
2) $6m^2+12mn+6n^2$, bunda $m=56, n=44$;
3) $-36a^3+4a^2b-\frac{1}{9}ab^2$, bunda $a=4, b=48$;
4) $-64a^3-8a^2b-\frac{1}{4}ab^2$, bunda $a=-6, b=84$.

418. Tenglamani yeching:
1) $x^2-36=0$; 3) $4x^2+4x+1=0$;
2) $\frac{1}{4}-x^2=0$; 4) $25-10x+x^2=0$.

419. Hisoblang:
1) $101^2-202\cdot81+81^2$; 3) $\frac{48^2+2\cdot48\cdot18+18^2}{48^2-18^2}$;
2) $37^2+126\cdot37+63^2$; 4) $\frac{85^2-17^2}{85^2+2\cdot85\cdot17+17^2}$.

420. Tushirib qoldirilgan shunday uchhadni topingki, tenglik bajarilsin:
1) $x^3+y^3=(x+y)(\ldots)$; 3) $x^3-y^3=(x-y)(\ldots)$;
2) $(x+y)^3=(x+y)(\ldots)$; 4) $(x-y)^3=(x-y)(\ldots)$.

421. Ko‘paytuvchilarga ajrating:
1) x^3-y^3; 3) x^3+27; 5) n^3-64; 7) $1-p^3$;
2) c^3+d^3; 4) a^3-27; 6) a^3+1; 8) $125-b^3$.

Ko‘paytuvchilarga ajrating **(422—424):**

422. 1) $27m^3-8$; | 2) $64-125y^3$; | 3) $125+\frac{1}{8}b^3$; | 4) $64y^3+\frac{1}{27}$.

423. 1) $8a^3+1$; 3) $\frac{1}{27}a^3+64b^6$;
2) $1+27b^3$; 4) $\frac{1}{8}a^6+125b^3$.

424. 1) $a^9 - b^3$; 2) $a^6 - b^6$; 3) $x^6 - 729$; 4) $64 - y^6$.

Ifodani qisqa ko‘paytirish formulalaridan foydalanib, ikkihad shaklida yozing **(425—426):**

425. 1) $(z+5)(z^2-5z+25)$;
2) $(y+2)(y^2-2y+4)$;
3) $(2x+3y)(4x^2-6xy+9y^2)$;
4) $(4c-5d)(16c^2+20cd+25d^2)$.

426. 1) $(10a^2-1)(100a^4+10a^2+1)$;
2) $(a^2b^2-5a)(a^4b^4+5a^3b^2+25a^2)$;
3) $\left(\frac{1}{5}m-n\right)\left(\frac{1}{25}m^2+\frac{1}{5}mn+n^2\right)$;
4) $\left(\frac{1}{2}x-\frac{1}{3}y\right)\left(\frac{1}{4}x^2+\frac{1}{6}xy+\frac{1}{9}y^2\right)$.

427. Ko‘paytuvchilarga ajrating:
1) $(8a^3-27b^3)-2a(4a^2-9b^2)$;
2) $(64a^3+125b^3)+5b(16a^2-25b^2)$;
3) $(a^3+b^3)+(a+b)^2$;
4) $(a^3-b^3)+(a-b)^2$.

428. Hisoblang:
1) $\frac{258^3-147^3}{258^2+258\cdot147+147^2}$;
2) $\frac{17,98^2-17,98\cdot32,02+32,02^2}{17,98^3+32,02^3}$.

429. Qavslar ichiga shunday hadlar yozingki, hosil bo‘lgan ifoda x ning barcha qiymatlarida ham o‘zgarmas bo‘lsin:
1) $(4x-7)^2+(3x+6)^2-(\ldots-\ldots)^2$;
2) $(17x-2)^2-(15x-6)^2-(\ldots+\ldots)^2$.

430. Tenglamani yeching:
1) $(x+2)(x^2-2x+4)-x(x-3)(x+3)=26$;
2) $(x-3)(x^2+3x+9)-x(x+4)(x-4)=21$;
3) $(2x-1)(4x^2+2x+1)-4x(2x^2-3)=23$;
4) $(4x+1)(16x^2-4x+1)-16x(4x^2-5)=17$.

Ko‘paytuvchilarga ajrating **(431—434):**

431. 1) $3a^3-3$; 2) y^3-y; 3) m^3n-mn^3; 4) $2a^3-2ab^2$.

432. 1) $x^4y^2-x^2y^4$; 3) $8-72x^6y^2$;
2) $7c^2d^2-63c^2b^2$; 4) $32a^4b-2a^2b$.

433. 1) $2a^2+4ab+2b^2$; 4) $8p^2-16p+8$;
2) $2m^2+2n^2-4mn$; 5) $27a^2b^2-18ab+3$;
3) $5x^2+10xy+5y^2$; 6) $12m^5n+24m^4n+12m^3n$.

434. 1) $2c^3+2d^3$; 3) $2cd^3-16c^4$; 5) $7x^2-56x^2y^3$;
2) $54x^3-16$; 4) $\frac{1}{8}a^2-a^5$; 6) $4a^2b+32a^5b$.

435. Hisoblang: $19,7^2-8,3^2+28\cdot 8,6$.

436. 1) Agar n — toq son bo‘lsa, $(n+2)^2-1$ ifodaning 8 ga;
2) ixtiyoriy natural son n da n^3+12n^2+23n ifodaning 6 ga bo‘linishini isbotlang.

Ko‘paytuvchilarga ajrating **(437—438):**

437. 1) $\left(a^2+2ab+b^2\right)-c^2$; 3) $1-a^2-2ab-b^2$;
2) $1-\left(x^2-2xy+y^2\right)$; 4) $4+\left(-x^2-2xy-y^2\right)$.

438. 1) a^2-b^2+a+b; 3) $x-y-x^2+y^2$; 5) $m^5-m^3+m^2-1$;
2) a^2-b^2-a-b; 4) x^3+x^2-x-1; 6) x^4+x^3+x+1.

439. 27^2-14^2 soni 13 ga bo‘linishini isbotlang.

440. n istalgan butun son bo‘lganda $(7n-2)^2-(2n-7)^2$ ifodaning qiymati 5 ga bo‘linishini; 9 ga bo‘linishini isbot qiling.

441. Tenglamani yeching:
1) $(x-3)\left(x^2+3x+9\right)-(3x-17)=x^3-12$;
2) $5x-\left(4-2x+x^2\right)(x+2)+x(x-1)(x+1)=0$.

442. Motorli qayiqning oqim bo‘yicha tezligi 18 km/soat, oqimga qarshi tezligi esa 14 km/soat. Daryo oqimining tezligini va qayiqning turg‘un suvdagi tezligini toping.

O‘zingizni tekshirib ko‘ring!

1. Ifodani standart ko‘phad ko‘rinishida tasvirlang:

$$(a-3)^2+(a-3)(a+3)+6a.$$

2. Ko‘paytuvchilarga ajrating:

1) $xy-2y$; 2) $16a^2-81$; 3) $3x^2-6x^3$;

4) $x^2-10x+25$; 5) $3(x-1)+y(x-1)$; 6) $2a^2-4ab+2b^2$.

3. Ko‘phadni ko‘paytuvchilarga ajrating va uning $a=1,\ b=-\frac{1}{3}$ bo‘lgandagi son qiymatini toping:

$$a^2-3ab+3a-9b.$$

IV bobga doir mashqlar

Ko‘paytuvchilarga ajrating **(443—447)**:

443. 1) $6(a+b)+(a+b)^2$; 3) $(a-b)+(b-a)^2$;

2) $4(x-y)+3(x-y)^2$; 4) $(a-b)^2-(b-a)$.

444. 1) $3(x+y)(x-y)+(x+y)^2$; 3) $5(a-b)^2-(a+b)(b-a)$;

2) $(x+y)^3-x(x+y)^2$; 4) $a(a-b)^2-(b-a)^2$.

445. 1) $(y+z)(12x^2+6x)+(y-z)(12x^2+6x)$;

2) $(y-z)(12x^2-6x)+(y-z)(12x^2+6x)$;

3) $(6x^2-3)+7x(6x^2-3)-4y(6x^2-3)$;

4) $2x(8x-4y)-3y(8x-4y)-(8x-4y)$.

446. 1) $18a^2-27ab+14ac-21bc$;

2) $10x^2+10xy+5x+5y$;

3) $35ax+24xy-20ay-42x^2$;

4) $48xz^2+32xy^2-15yz^2-10y^3$.

447. 1) $16ab^2-5b^2c-10c^3+32ac^2$;

2) $6mnk^2+15m^2k-14n^3k-35mn^2$;

3) $-28ac+35c^2-10cx+8ax$;

4) $-24bx-15c^2+40bc+9cx$.

448. Ifodani soddalashtiring:

1) $(2x-1)^2-2(2x-3)^2+17$;

2) $(3x+2)^2-2(x-1)^2-7x^2$;

3) $24y^2-(7y-2)^2+(5y-3)(5y+1)$;

4) $(3y+1)(2y-3)+(2y-3)^2-10y^2$.

449. Ikkita ketma-ket natural son kvadratlari ayirmasining moduli toq son bo‘lishini isbotlang.

450. Kasrni qisqartiring:

1) $\frac{53^2-27^2}{79^2-51^2}$;

2) $\frac{38^2-17^2}{47^2-19^2}$;

3) $\frac{49^2-2\cdot49\cdot29+29^2}{49^2-19^2}$;

4) $\frac{47^2-3^2}{27^2+2\cdot27\cdot13+13^2}$.

451. x va y ning istalgan qiymatlarida tenglik to‘g‘ri bo‘lishini isbotlang: $(x+y)(x^2-y^2)=(x-y)(x+y)^2$.

№ 8 *1) Oiladagi 6 ta qizning har birining akasi bor. Shu oilada nechta farzand bor?*

2) Muhammadjonning akalari qancha bo‘lsa, opalari ham shuncha. Katta opasining ukalari soni singillari sonidan 2 marta ko‘p. Shu oilada nechta o‘g‘il, nechta qiz bor?

IV bobga doir sinov mashqlari — testlar

1. Umumiy ko‘paytuvchini qavsdan tashqariga chiqaring: $24a^3b^2 - 30a^2b^3$.

A) $6a^2b^2(4a - 5b)$; B) $6ab(4a^2b - 5ab^2)$;

C) $6a^2(4ab^2 - 5b^3)$; D) $6b^2(4a^3 - 5a^2)$.

2. Ko‘paytuvchilarga ajrating: $5(a-b) + a^2(a-b) - 3(b-a)$.

A) $(a-b)(a^2+2)$; B) $(a-b)(a^2-8)$;

C) $(a-b)(8-a^2)$; D) $(a-b)(a^2+8)$.

3. Ko‘paytuvchilarga ajrating: $4a(x-y) + 4az + 7b(y-x-z)$.

A) $(x-y+z)(4a-7b)$; B) $(y-x-z)(7b+4a)$;

C) $(x-y-z)(4a-7b)$; D) $-(x-y+z)(4a+7b)$.

4. Hisoblang: $16,9^2 - 16,9 \cdot 3,7 - 16,9 \cdot 3,2$.

A) 169; B) 1,69; C) 16,9; D) –1,69.

5. Ko‘paytuvchilarga ajrating: $ax + bx - 3ay - 3by$.

A) $(a+b)(x+3y)$; B) $(a-b)(x+3y)$;

C) $(a-b)(x-3y)$; D) $(a+b)(x-3y)$.

6. Ko‘paytuvchilarga ajrating: $7a(5a-3b) - 10a + 6b$.

A) $(5a+3b)(7a-2)$; B) $(3b-5a)(7a+2)$;

C) $(5a-3b)(7a-2)$; D) $(5a-3b)(7a+2)$.

7. Tenglamani yeching: $(3x+2)^2 - (3x-4)^2 = 132$.

A) 4; B) 3; C) –5; D –4.

8. Ko‘paytuvchilarga ajrating: $8a^3 - 27b^3$.

A) $(2a-3b)^2(2a+3b)$; B) $(2a+3b)^2 \cdot (2a-3b)$;

C) $(2a)^3 - (3b)^3$; D) $(2a-3b)(4a^2+6ab+9b^2)$.

9. Hisoblang: $(53^3 + 47^3) : (53^2 - 53 \cdot 47 + 47^2)$.

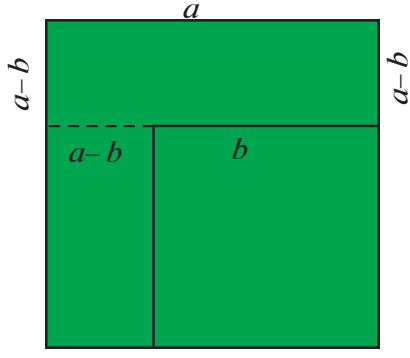
A) 6; B) 100; C) 600; D) $53^2 + 47^2$.

Tarixiy ma’lumotlar

Al-Koshiyning „Arifmetika kaliti“ asarida ikkihadni ixtiyoriy natural darajaga ko‘tarish qoidalari berilgan.

Turli algebraik formulalarni isbotlashda, tenglamalarni yechishda geometrik mulohazalardan foydalanish qadimgi Xitoy, Yunoniston, Hindiston, O‘rta Osiyo matematiklari asarlarida uchraydi.

Ular $(a+b)^2=a^2+2ab+b^2$, $(a-b)^2=a^2-2ab+b^2$, $a^2-b^2=(a-b)\times(a+b)$ (yoki $(a^2-b^2)=(a-b)^2+2b(a-b)$) kabi ayniyatlarni geometrik usulda isbotlaganlar. Masalan, $a^2-b^2=(a-b)(a+b)$ formulani isbotlashga shunday yondashilgan: tomoni a ga teng kvadratdan tomoni b ga teng kvadratni qirqib olinsa, qolgan shaklning yuzi: $a(a-b)+b(a-b)=(a-b)(a+b)$ ga, yoki baribir, $(a-b)^2+2b(a-b)$ ga teng bo‘lishi 21- rasmdan ravshan ko‘rinib turibdi.



21- rasm.

Demak, $a^2-b^2=(a-b)(a+b)$ formula to‘g‘ri.

To‘g‘ri burchakli uchburchakning tomonlarini butun (yoki ratsional) sonlarda ifodalash uchun Xitoy matematiklari miloddan avvalgi birinchi ming yillardayoq

$$\left(\frac{p^2-q^2}{2}\right)^2+(pq)^2=\left(\frac{p^2+q^2}{2}\right)^2$$

tenglikdan foydalanganlar.

ALGEBRAIK KASRLAR

-§ Algebraik kasr. Kasrlarni qisqartirish

- masala. Katerning turg‘un suvdagi tezligi soatiga a ki-
trga, daryo oqimining tezligi soatiga b kilometrga teng.
rning daryo oqimi bo‘yicha harakat tezligi uning daryo
iga qarshi harakat tezligidan necha marta ortiq?

Katerning daryo oqimi bo‘yicha tezligi soatiga $(a+b)$ ki-
trga teng; oqimga qarshi tezligi soatiga $(a-b)$ kilometrga
. Shuning uchun daryo oqimi bo‘yicha harakat tezligi
ga qarshi harakat tezligidan

$$\frac{a+b}{a-b}$$

a ortiq bo‘ladi. ▲

$\frac{+b}{-b}$ ifoda *algebraik kasr* deyiladi. Bu kasrning surati $a+b$,
aji esa $a-b$.

Umuman, *surat va maxraji algebraik ifodalar bo‘lgan kasr*
raik kasr deyiladi.

lgebraik kasrlarga doir yana bir necha misollar keltiramiz:

$$\frac{a}{b};\quad \frac{2}{x+y};\quad \frac{a-b}{c};\quad \frac{x(b+c)}{y(a-c)}.$$

gar algebraik kasrga kiruvchi harflar o‘rniga biror sonlar
lsa, u holda zarur hisoblashlar bajarilgandan keyin shu
raik kasrning *son qiymati* hosil bo‘ladi.

Masalan, $a=10$, $b=8$ bo‘lganda $\frac{a+b}{a-b}$ algebraik kasrning

qiymati $\frac{10+8}{10-8}=\frac{18}{2}=9$ ga teng bo‘ladi.

($a \neq b$) istalgan sonlarni qo‘yish mumkin, chunki $a = b$ ganda kasrning maxraji nolga aylanadi, nolga bo‘lish mumkin emas.

Bundan keyin algebraik kasrga kiruvchi harflar yo‘l q ladigan (joiz) qiymatlarnigina, ya’ni shu kasrning ma nolga teng bo‘lmaydigan qiymatlarnigina qabul qiladi, shartlashamiz.

Masalan, $\frac{a}{a(a-1)}$ kasr uchun joiz qiymatlar a ning va $a=1$ dan boshqa barcha qiymatlari bo‘ladi.

Kasrning asosiy xossasini bunday yozish mumkin:

$$\frac{a}{b} = \frac{ma}{mb},$$

bu yerda $b \neq 0$, $m \neq 0$.

Bu xossa kasrning surat va maxrajini bir xil algebraik daga ko‘paytirilsa yoki bo‘linsa, unga teng kasr hosil bo‘li bildiradi, masalan:

$$\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}, \quad \frac{a+b}{b} = \frac{(a+b) \cdot c}{bc}.$$

Kasrning asosiy xossasidan foydalanib, algebraik kasrni u surat va maxrajiga bir vaqtda kiruvchi umumiy ko‘paytuv qisqartirish mumkin, masalan:

$$\frac{a(b+c)}{a(b-c)} = \frac{b+c}{b-c}, \quad \frac{(a+b)c}{(a+b)d} = \frac{c}{d}.$$

Kasrlarni soddalashtirish uchun avval ularning sura maxrajining umumiy ko‘paytuvchisini ajratib olish kerak doir misollar keltiramiz.

2- masala. Kasrlarni qisqartiring:

1) $\frac{12a^2b}{4ab^2}$; 2) $\frac{m^2-n^2}{m^2+mn}$.

$$\frac{12a^2b}{4ab^2}=\frac{4ab\cdot 3a}{4ab\cdot b}=\frac{3a}{b}.$$

) m^2-n^2 va m^2+mn ko‘phadlar $m+n$ umumiy ko‘paytuv-
a ega, chunki $m^2-n^2=(m+n)(m-n)$, $m^2+mn=m(m+n)$.
ning surat va maxrajini $m+n$ ga bo‘lamiz:

$$\frac{m^2-n^2}{m^2+mn}=\frac{(m+n)(m-n)}{m(m+n)}=\frac{m-n}{m}. \blacktriangle$$

Kasrlarni qisqartirish uchun bu kasrlarning surat va maxrajini ularning umumiy ko‘paytuvchisiga bo‘lish kerak.

Agar $\frac{a}{b}$ kasrning surat yoki maxrajidagi ishora qarama-qarshisiga o‘zgartirilsa, u holda berilgan kasrga qarama-qarshi kasr hosil bo‘lishini ta’kidlab o‘tamiz:

$$\frac{-a}{b}=-\frac{a}{b};\quad \frac{a}{-b}=-\frac{a}{b}.$$

Masalan, $\frac{-3}{7}=-\frac{3}{7}$; $\frac{-a}{1-a}=-\frac{a}{1-a}=\frac{a}{a-1}$.

- masala. $\frac{3a(y-x)}{a^2(x-y)}$ kasrni qisqartiring:

$\triangle$ $\frac{3a(y-x)}{a^2(x-y)}=\frac{-3a(x-y)}{a^2(x-y)}=\frac{-3}{a}=-\frac{3}{a}$. $\blacktriangle$

Mashqlar

Surati x va y sonlarning ko‘paytmasiga, maxraji esa ularning yig‘indisiga teng algebraik kasrni yozing.

Surati p va q sonlarning ayirmasiga, maxraji esa ularning ko‘paytmasiga teng bo‘lgan algebraik kasrni yozing.

gebraik kasrni yozing.

455. Surati c va d sonlar kublarining yig‘indisiga, maxraj shu sonlar ko‘paytmasining ikkilanganiga teng bo‘lga gebraik kasrni yozing.

456. Algebraik kasrning son qiymatini toping:

1) $\frac{1}{a}$, bunda $a = 2\frac{3}{5}$;

2) $\frac{b+1}{b-1}$, bunda $b = 1{,}5$;

3) $\frac{a^2+1}{2a}$, bunda $a = -3$;

4) $\frac{a-b}{a+2b}$, bunda $a = 16, b =$

5) $\frac{5a+b^2}{a^2-5b}$, bunda $a = 2, b =$

6) $\frac{-7ab}{3b^2-a^3}$, bunda $a = 3, b =$

457. 1) $S = vt$ formuladan v ni; 2) $p = \frac{m}{V}$ formuladan

3) $C = 2\pi R$ formuladan R ni;

4) $P = 2(a+b)$ formuladan a ni toping.

458. Har bir yuk mashinasiga a tonnadan kartoshka yu mumkin bo‘lsa, har birida p kilogrammdan karto bo‘lgan n qop kartoshkani tashib ketish uchun nechta mashinasi (x) kerak bo‘ladi? x ni $n = 90$, $p = 50$, a = bo‘lganda toping.

459. Mashina soatiga o‘rtacha c metr linoleum ishlab chiqa Agar mashina kuniga n soatdan ishlasa, u a metr leumni necha kunda ishlab chiqaradi? Izlanayotgan v t bilan belgilab, t ni $c = 47$, $a = 11280$ va $n = 16$ bo‘lg toping.

460. Berilgan ikkita kasrning tengligini ko‘rsating:

1) $\frac{6}{7}$ va $\frac{18}{21}$;

2) $\frac{-3}{5}$ va $\frac{27}{-45}$;

3) $\frac{2}{3}$ va $\frac{2a}{3a}$;

4) $\frac{2a}{7b}$ va $\frac{2a^2b}{7ab^2}$;

5) $\frac{m-n}{m+n}$ va $\frac{m^2-n}{(m+n)}$

6) $\frac{a+3b}{c}$ va $\frac{(a+3b}{c^2}$

1) $\frac{}{-56}$; 2) $\frac{}{-80}$; 3) $\frac{}{55}$; 4) $\frac{}{-14}$.

1) $\frac{12a}{20}$; 2) $\frac{2c}{3c}$; 3) $\frac{7b}{21b}$; 4) $\frac{4ab}{8ac}$; 5) $\frac{a^2}{2a}$; 6) $\frac{5x}{x^3y}$.

1) $\frac{a^2}{a^3}$; 2) $\frac{b^3}{b^7}$; 3) $\frac{a^5}{a^4}$; 4) $\frac{b^6}{b^4}$.

Kasrni qisqartiring **(464 — 474):**

1) $\frac{6ab}{4a}$; 3) $\frac{a^4b}{ab^3}$; 5) $\frac{12a^4b^2}{18a^3b^3}$;

2) $\frac{14c}{49c}$; 4) $\frac{3a^2b}{9a^3}$; 6) $\frac{25a^3bc^2}{125ac^3}$.

1) $\frac{4(m+n)}{5(m+n)}$; 3) $\frac{2b(m-n)}{8b(m-n)(m-n)}$; 5) $\frac{2(a-b)}{b-a}$;

2) $\frac{7a(a-b)}{5(a-b)}$; 4) $\frac{3a(a+b)}{9a(a+b)(a-b)}$; 6) $\frac{5(x-y)}{15(y-x)}$.

1) $\frac{(a-b)^2}{a-b}$; 3) $\frac{m-n}{(n-m)^2}$; 5) $\frac{3m(1-x)^2}{9m^2(x-1)^2}$;

2) $\frac{m+n}{(m+n)^4}$; 4) $\frac{(2x-3y)^2}{3y-2x}$; 6) $\frac{8a^2b(a-b)}{4a^3b(b-a)^2}$.

1) $\frac{3x+3y}{6c}$; 3) $\frac{2a+2b}{4a-4b}$; 5) $\frac{ac-bc}{ac+bc}$;

2) $\frac{8a}{4m-4n}$; 4) $\frac{12a-3}{6a+9}$; 6) $\frac{a+ab}{a-ab}$.

1) $\frac{a^2}{a^2+ab}$; 3) $\frac{7a+14b}{3a+6b}$; 5) $\frac{3a-6b}{12b-6a}$;

2) $\frac{pq^3}{p^2q-pq^2}$; 4) $\frac{2m^2-mn}{2mn-n^2}$; 6) $\frac{x^2-2xy}{2y^2-xy}$.

3) $\frac{m^3-3m^2n}{3m^2n-3m^3}$; 4) $\frac{a^3-2a^2b}{2a^3b^2-a^4b}$.

470. 1) $\frac{a^2-b^2}{a+b}$; 3) $\frac{4c^2-9x^2}{2c-3x}$; 5) $\frac{3a(a-b)}{6a^2(b-a)}$;

2) $\frac{a-b}{a^2-b^2}$; 4) $\frac{25-x^2}{5-x}$; 6) $\frac{5a(c^2-4)}{10a^2(2-c)}$.

471. 1) $\frac{8-3c}{9c^2-64}$; 3) $\frac{2y-10}{25-y^2}$; 5) $\frac{b^2-c^2}{b^4n-c^4n}$;

2) $\frac{100-49b^2}{7b+10}$; 4) $\frac{5y-y^2}{25-y^2}$; 6) $\frac{5a^3b+5ab^3}{a^4-b^4}$.

472. 1) $\frac{d^2-6d+9}{d-3}$; | 2) $\frac{b+7}{b^2+14b+49}$; | 3) $\frac{9-6a+a^2}{3-a}$; | 4) $\frac{1-2}{1-4p+}$

473. 1) $\frac{4y^2-4y+1}{4y^2-1}$; 3) $\frac{3a^2-6ab+3b^2}{6a^2-6b^2}$;

2) $\frac{16a^2-1}{16a^2-8a+1}$; 4) $\frac{50m^2+100mn+50n^2}{15m^2-15n^2}$.

474. 1) $\frac{1-a^2}{(a-1)^2}$; 3) $\frac{4y^2-4y+1}{2-4y}$;

2) $\frac{(m-n)^2}{n-m}$; 4) $\frac{5-2x}{4x^2-20x+25}$.

475. Kasrni qisqartiring:

1) $\frac{9c^2-16}{16-24c+9c^2}$; 4) $\frac{36c-c^3}{c^3+12c^2+36c}$;

2) $\frac{16x^2-24xy+9y^2}{9y^2-16x^2}$; 5) $\frac{25b-49b^3}{49b^3-70b^2+25b}$;

3) $\frac{4x^2-4xy+y^2}{y^2-4x^2}$; 6) $\frac{4b^2-12bc+9c^2}{-2ab+3ac}$.

1) $\frac{2a-128a}{(2a^2+8a+32)(a^4-4a^3)}$;

2) $\frac{2a^4+3a^3+2a+3}{(a^2-a+1)(2a+3)}$;

3) $\frac{3a^3+ab^2-6a^2b-2b^3}{9a^5-ab^4-18a^4b+2b^5}$;

4) $\frac{3ac^2+3bc^2-3ab^2-3b^3}{6ac^2+6bc^2-6ab^2-6b^3}$.

- §. *Kasrlarni umumiy maxrajga keltirish*

ddiy kasrlarni qo‘shishda avval kasrlarni umumiy max- keltirib olinadi. Masalan, $\frac{1}{4}$, $\frac{3}{25}$, $\frac{7}{10}$ kasrlar uchun umu- maxraj 100 soni bo‘ladi, bu son 4, 25, 10 sonlarining kichik umumiy karralisidir.

Algebraik kasrlarning umumiy maxraji shu kasrlar maxrajlarining eng kichik umumiy karralisidir. Kasrlarni umumiy maxrajga keltirishda kasrning asosiy xossasidan foydalaniladi.

- masala. $\frac{m}{3a^2b}$, $\frac{n}{6ab^2}$ va $\frac{p}{4ac}$ algebraik kasrlarni umumiy rajga keltiring.

Berilgan kasrlarning umumiy maxraji har bir kasrning rajiga bo‘linishi kerak. Demak, u 3 ga, 6 ga, 4 ga, ya’ni a; a^2 ga, a ga va a ga, ya’ni a^2 ga; b ga va b^2 ga, ya’ni b^2 ga; bo‘linishi kerak.

hunday qilib, kasrlarning umumiy maxraji 12, a^2, b^2 va c aytuvchilarni o‘z ichiga olishi kerak. Umumiy maxraj da $12a^2b^2c$ ko‘paytmani olish lozim bo‘ladi. Bu umumiy rajni birinchi kasrning maxrajiga bo‘lib, uning surat va rajini ko‘paytirish kerak bo‘lgan birhadni topamiz. Bu ad berilgan *kasrning qo‘shimcha ko‘paytuvchisi* deyiladi. chi kasr uchun bunday birhad $4bc$ ga teng. Xuddi shunday bilan ikkinchi va uchinchi kasrlar uchun qo‘shimcha aytuvchilarni topamiz: $2ac$ va $3ab^2$.

$12a^2b^2c$ umumiy maxrajga keltiramiz:

$$\frac{m}{3a^2b}=\frac{4mbc}{12a^2b^2c},\quad \frac{n}{6ab^2}=\frac{2nac}{12a^2b^2c},\quad \frac{p}{4ac}=\frac{3pab^2}{12a^2b^2c}.\ \blacktriangle$$

2- masala. Kasrlarni umumiy maxrajga keltiring:

$$\frac{a}{x^2-y^2};\quad \frac{b}{2x^2-4xy+2y^2};\quad \frac{c}{3x^2+6xy+3y^2}.$$

△ Kasrlarning maxrajini ko‘paytuvchilarga ajratamiz:

$$x^2-y^2=(x-y)(x+y);$$

$$2x^2-4xy+2y^2=2\left(x^2-2xy+y^2\right)=2(x-y)^2;$$

$$3x^2+6xy+3y^2=3\left(x^2+2xy+y^2\right)=3(x+y)^2.$$

Umumiy maxraj berilgan kasrlarning har birining maxr
bo‘linishi kerak.

Umumiy maxraj birinchi kasrning maxrajiga bo‘li
uchun uning tarkibida $(x-y)(x+y)$ ko‘paytma bo‘lishi ke

So‘ngra, umumiy maxraj ikkinchi kasrning maxrajiga b
nishi kerak va shuning uchun unda $2(x-y)^2$ ko‘paytuvchi bo
kerak. Demak, birinchi kasr maxrajiga $2(x-y)$ ko‘paytuv
yozib qo‘yish kerak, ya’ni umumiy maxraj tarkibida

$$2(x-y)^2(x+y)$$

ko‘paytma bo‘lishi lozim.

Umumiy maxraj uchinchi kasrning $3(x+y)^2$ maxrajiga
linishi uchun hosil qilingan ko‘paytmaga $3(x+y)$ ko‘pay
chini yozib qo‘yish kerak. Demak, uchala kasrning umu
maxraji

$$6(x-y)^2(x+y)^2$$

ga teng bo‘ladi.

Kasrlarni umumiy maxrajga keltirish uchun ularning
va maxrajini qo‘shimcha ko‘paytuvchilarga ko‘paytirish k
ular esa umumiy maxrajni har bir kasrning maxrajiga bo

$$6(x-y)(x+y),\ \ 3(x+y)^2,\ \ 2(x-y)^2.$$

ak, berilgan kasrlarni bunday yozib olish mumkin:

$$\frac{a}{-y^2}=\frac{6a\,(x-y)\,(x+y)}{6\,(x-y)^2\,(x+y)^2};\qquad \frac{b}{2x^2-4xy+2y^2}=\frac{3b\,(x+y)^2}{6\,(x-y)^2\,(x+y)^2};$$

$$\frac{c}{3x^2+6xy+3y^2}=\frac{2c\,(x-y)^2}{6\,(x-y)^2\,(x+y)^2}.\ \blacktriangle$$

Algebraik kasrlarni umumiy maxrajga keltirish uchun:

1) berilgan kasrlarning umumiy maxrajini topish;

2) har bir kasr uchun qo‘shimcha ko‘paytuvchini topish;

3) har bir kasrning suratini uning qo‘shimcha ko‘paytuvchisiga ko‘paytirish;

4) har bir kasrni topilgan surat va umumiy maxraj bilan yozish kerak.

Mashqlar

Quyidagi mashqlarda kasrlarni umumiy maxrajga keltiring **(—484):**

1) $\frac{1}{2}$ va $\frac{2}{3}$;

2) $\frac{1}{a}$ va $\frac{2}{b}$;

3) $\frac{5}{7}$ va $\frac{3}{14}$;

4) $\frac{a}{b}$ va $\frac{a}{2b}$;

5) $\frac{x}{2y}$ va $\frac{x}{3y}$;

6) $\frac{8}{15}$ va $\frac{5}{12}$.

1) $\frac{3}{4a}$, $\frac{1}{5b}$ va $\frac{7}{20ab}$;

2) $\frac{3x}{4y}$, $\frac{6}{xy}$ va $\frac{4y}{3x}$;

3) $\frac{7}{a^2}$ va $\frac{8}{a^3}$;

4) $\frac{a}{2x}$ va $\frac{b}{4x^3}$.

1) a va $\frac{b^2}{a}$;

2) $3b$ va $\frac{a^2}{2b}$;

3) a^2 va $\frac{c}{2ab}$;

4) $\frac{b}{3a}$, $\frac{3c}{2b}$ va ab.

2) $\frac{1}{6b^2}$, $\frac{a^2+b^2}{9a^2b^2}$ va $\frac{3-a^2}{18ab^2}$;

4) $\frac{7}{20x^4y}$, $\frac{31}{6xy^3}$ va $\frac{4}{3x^2y^4}$.

481. 1) $\frac{3}{x+y}$ va $\frac{5}{x}$;

3) $\frac{7x}{2(x-1)}$ va $\frac{5x}{x-1}$;

2) $\frac{6}{a-1}$ va $\frac{2}{a}$;

4) $\frac{2a^2}{3(a+1)}$ va $\frac{5a^2}{4(a+1)}$.

482. 1) $\frac{1}{x-y}$ va $\frac{1}{x+y}$;

3) $\frac{5x}{2x-2}$ va $\frac{3}{4x-4}$.

2) $\frac{7a}{3x-y}$ va $\frac{6b}{3x+y}$;

4) $\frac{3x}{4x+4y}$ va $\frac{x}{8x+8y}$;

483. 1) $\frac{3b}{b-2}$ va $\frac{4}{b^2-4}$;

3) $\frac{1}{1-a}$, $\frac{2a}{1+a}$ va $\frac{a^2}{1-a^2}$;

2) $\frac{7a}{x^2-9}$ va $\frac{a}{x+3}$;

4) $\frac{6x}{x-y}$, $\frac{7xy}{x+y}$ va $\frac{3}{x^2-y^2}$.

484. 1) $\frac{m}{2m+2n}$, $\frac{n}{8m-8n}$ va $\frac{mn}{6m^2-6n^2}$;

2) $\frac{2c}{5b-5c}$, $\frac{3a^2}{35b^2-35c^2}$ va $\frac{7b}{14b+14c}$;

3) $\frac{1}{a^2-4b^2}$, $\frac{1}{3a^2+6ab}$ va $\frac{1}{2ab-a^2}$;

4) $\frac{5}{4x-4}$, $\frac{4x}{1-x^2}$ va $\frac{1}{3x^2+3x}$.

№ 9 *Bir qurt yerdan daraxtning uchiga chiqmoqchi bo'*
Daraxt bo'ylab kechasi u 2 m balandlikka chiq
kunduzi esa 1 m pastga tushar ekan. 9-kecha
daraxtning uchiga chiqib olibdi. Daraxtning bala
necha metr ekan?

ir xil maxrajli kasrlarni qo'shish va ayirish qoidalarini ay yozish mumkin:

$$\frac{a}{m}+\frac{b}{m}=\frac{a+b}{m};$$

$$\frac{a}{m}-\frac{b}{m}=\frac{a-b}{m}.$$

- masala. $\frac{a-b}{a+b}$, $\frac{2a-b}{a+b}$ va $\frac{a-2b}{a+b}$ kasrlarni qo'shing.

$$\frac{a-b}{a+b}+\frac{2a-b}{a+b}+\frac{a-2b}{a+b}=\frac{a-b+2a-b+a-2b}{a+b}=\frac{4a-4b}{a+b}=\frac{4\left(a-b\right)}{a+b}.$$ ▲

- masala. $\frac{a^2}{a+b}$ va $\frac{b^2}{a+b}$ kasrlarning ayirmasini toping.

△ $$\frac{a^2}{a+b}-\frac{b^2}{a+b}=\frac{a^2-b^2}{a+b}=\frac{\left(a+b\right)\left(a-b\right)}{a+b}=a-b.$$ ▲

Har xil maxrajli kasrlarni qo'shish va ayirish uchun bu kasrlarni umumiy maxrajga keltirish va bir xil maxrajli kasrlarni qo'shish yoki ayirish qoidasidan foydalanish kerak.

- masala. $\frac{1}{a^3}$, $\frac{1}{2a^2b}$ va $\frac{1}{3ab^2}$ kasrlarni qo'shing.

Berilgan kasrlarning umumiy maxraji $6a^3b^2$ ko'paytma di. Demak,

$$\frac{1}{a^3}+\frac{1}{2a^2b}+\frac{1}{3ab^2}=\frac{6b^2}{6a^3b^2}+\frac{3ab}{6a^3b^2}+\frac{2a^2}{6a^3b^2}=\frac{2a^2+3ab+6b^2}{6a^3b^2}.$$ ▲

- masala. $\frac{a}{3b^2c}$ va $\frac{c}{15ab^2}$ kasrlarning ayirmasini toping.

△ $$\frac{a}{3b^2c}-\frac{c}{15ab^2}=\frac{5a^2}{15ab^2c}-\frac{c^2}{15ab^2c}=\frac{5a^2-c^2}{15ab^2c}.$$ ▲

△ Kasrlarning maxrajlarida turgan ko‘phadlarni ko‘
tuvchilarga ajratamiz:

$$x^2-x=x(x-1),\ x^2-1=(x-1)(x+1).$$

Kasrlarning umumiy maxraji $x(x-1)(x+1)$ ko‘paytma bo
Kasrlarni umumiy maxrajga keltirib, topamiz:

$$\frac{1}{x^2-x}+\frac{3}{x^2-1}=\frac{1}{x(x-1)}+\frac{3}{(x-1)(x+1)}=\frac{x+1}{x(x^2-1)}+\frac{3x}{x(x^2-1)}=$$

$$=\frac{x+1+3x}{x(x^2-1)}=\frac{4x+1}{x(x^2-1)}.\ ▲$$

Turli maxrajli kasrlarni qo‘shish va ayirishni u
tartibda bajarish mumkin:

1) kasrlarning umumiy maxraji topiladi;
2) kasrlarni umumiy maxrajga keltiriladi;
3) hosil bo‘lgan kasrlarni qo‘shiladi;
4) mumkin bo‘lsa, natijani soddalashtiriladi.

6- masala. $\frac{1}{a^2+4a+4}-\frac{4}{a^4+4a^3+4a^2}+\frac{4}{a^3+2a^2}$ ifodaning qiymatini $a=0{,}5$ bo‘lganda hisoblang.

△ Berilgan ifodani quyidagicha almashtirish mumkin:

$$\frac{1}{(a+2)^2}-\frac{4}{a^2(a^2+4a+4)}+\frac{4}{a^2(a+2)}=\frac{1}{(a+2)^2}-\frac{4}{a^2(a+2)^2}+\frac{4}{a^2(a+2}$$

$$=\frac{a^2-4+4(a+2)}{a^2(a+2)^2}=\frac{a^2+4a+4}{a^2(a+2)^2}=\frac{1}{a^2}.$$

Demak, ifodaning izlanayotgan son qiymati:

$$\frac{1}{0{,}5^2}=\frac{1}{0{,}25}=\frac{100}{25}=4.\ ▲$$

Kasrlarning yig‘indisini (ayirmasini) toping **(485—491):**

1) $\frac{p}{q^2}+\frac{3p}{q^2}$;

2) $\frac{8a}{b^3}-\frac{3a}{b^3}$;

3) $\frac{a}{a+b}+\frac{c}{a+b}$;

4) $\frac{x}{n+a}-\frac{y}{n+a}$.

1) $\frac{c+d}{2a}+\frac{2c-d}{2a}$;

2) $\frac{a+2b}{3c^2}+\frac{5a-2b}{3c^2}$;

3) $\frac{a+b}{2c}-\frac{a-b}{2c}$;

4) $\frac{10a-b}{a^3}-\frac{3a-b}{a^3}$;

5) $\frac{(1+b)^2}{5d}+\frac{(1-b)^2}{5d}$;

6) $\frac{(2+a)^2}{a^2b}-\frac{(2-a)^2}{a^2b}$.

1) $\frac{2}{5}+\frac{3}{7}$;

2) $\frac{4}{7}-\frac{5}{28}$;

3) $\frac{2}{3a}+\frac{1}{a}$;

4) $\frac{1}{b}-\frac{2}{5b}$;

5) $\frac{c}{15a}+\frac{d}{3}$;

6) $\frac{a}{4}-\frac{b}{12d}$.

1) $\frac{m}{2}-\frac{1}{n}$;

2) $\frac{3}{a}+\frac{b}{5}$;

3) $5-\frac{1}{a}$;

4) $\frac{2}{b}+7$.

1) $5-\frac{2}{b}+\frac{3}{b^2}$;

2) $\frac{2}{c}+4-\frac{3}{c^2}$;

3) $d-\frac{c}{d}+\frac{c^2}{d^2}$;

4) $\frac{m}{n}-k+\frac{m^2}{n^2}$.

1) $\frac{1}{ab}+\frac{1}{bc}$;

2) $\frac{1}{mn}-\frac{1}{mk}$;

3) $\frac{a}{bc}-\frac{a}{bd}$;

4) $\frac{b}{ac}+\frac{b}{cd}$;

5) $\frac{3}{m^2}+\frac{4}{mn}$;

6) $\frac{2}{mn}-\frac{3}{n^3}$.

1) $\frac{3c}{4a^3b}+\frac{5d}{6ab^3}$;

2) $\frac{2a}{9b^4}-\frac{7c}{6a^3b}$;

3) $\frac{2}{3y^3}-\frac{1}{6x^2y}+\frac{5}{12xy^2}$;

4) $\frac{5}{7x^2y}-\frac{3}{4xy^2}+\frac{11}{14x^2y^2}$;

5) $\frac{a}{b^2}+\frac{b}{c^2}+\frac{c}{a^2}$;

6) $\frac{b}{c}+\frac{b}{c^2d}+\frac{b}{cd^2}$.

492. 1) $\frac{2x}{3(a-b)}+\frac{x}{a-b}$;

2) $\frac{7x}{2(x-1)}-\frac{5x}{x-1}$;

3) $\frac{2a^2}{3(a+1)}+\frac{5a^2}{4(a+1)}$;

4) $\frac{4y}{5(y-3)}-\frac{5x}{2(y-3)}$.

493. 1) $\frac{5}{2x-2}+\frac{3}{4x-4}$;

2) $\frac{7}{5b+5}-\frac{3}{10b+10}$;

3) $\frac{a}{3a+3b}-\frac{2a}{6a+6b}$;

4) $\frac{3x}{4x+4y}-\frac{x}{8x+8y}$.

494. 1) $\frac{3}{a^2+a}+\frac{5a}{ab+b}$;

2) $\frac{5b}{ax+ay}-\frac{2a}{bx+by}$;

3) $\frac{y+a}{b^2+ba}+\frac{y-b}{ab+a^2}$;

4) $\frac{y-b}{a^2-ab}-\frac{y-a}{ab-b^2}$.

495. 1) $\frac{3}{x+y}-\frac{5}{x}$;

2) $\frac{6}{a}-\frac{10}{a-1}$;

3) $\frac{1}{x(x-3)}+\frac{1}{x(x+3)}$;

4) $\frac{4}{5(a-b)}-\frac{7}{8(a+b)}$.

496. 1) $\frac{a}{1-b^2}+\frac{1}{1+b}$;

2) $\frac{2}{x^2-9}+\frac{1}{x+3}$;

3) $\frac{5+p^2}{p^2-36}-\frac{p}{6+p}$;

4) $\frac{2x}{x-4}-\frac{5x-2}{x^2-16}$.

497. 1) $\frac{2x}{x-4}-\frac{5x-2}{16-x^2}$;

2) $\frac{12n-5}{n^2-49}+\frac{6}{7-n}$;

3) $\frac{c^2-8}{2c+3}-\frac{16c-2c^3}{9-4c^2}$;

4) $\frac{21y^2+1}{1-9y^2}-\frac{y}{3y-1}$.

498. 1) $\frac{3}{a+2}+\frac{2a}{(a+2)^2}$;

2) $\frac{a}{(3a+1)^2}+\frac{4}{3a+1}$.

499. 1) $\frac{2y+8}{y^2-4y+4}-\frac{7}{y-2}$;

4) $\frac{4}{(m-n)^2}-\frac{7}{n-m}$;

3) $\frac{7}{(a-b)^2}-\frac{5}{b-a}$; 6) $\frac{1}{x^2-6x+9}+\frac{1}{(x+3)^2}$.

1) $a+\frac{a}{a-1}$; 2) $b-\frac{b}{b-2}$; 3) $c+1-\frac{c^2}{c-1}$; 4) $\frac{a^2}{a+1}-a+1$.

1) $\frac{7}{a+b}+\frac{8}{a-b}-\frac{16b}{a^2-b^2}$; 3) $\frac{3}{a+3}+\frac{2}{3-a}-\frac{6}{a^2-9}$;

2) $\frac{6x}{x^2-y^2}-\frac{3}{x-y}-\frac{4}{x+y}$; 4) $\frac{3}{4a^2-9}-\frac{8}{2a+3}-\frac{7}{3-2a}$.

1) $\frac{a+b}{a}-\frac{a}{a-b}-\frac{b}{a^2-ab}$; 4) $\frac{7}{m}-\frac{4}{m-2n}-\frac{m-n}{4n^2-m^2}$;

2) $\frac{5b-1}{3b^2-3}+\frac{b+2}{2b+2}-\frac{b+1}{b-1}$; 5) $x-\frac{xy}{x+y}-\frac{x^3}{x^2-y^2}$;

3) $\frac{6a}{9a^2-1}+\frac{3a+1}{3-9a}+\frac{3a-1}{6a+2}$; 6) $a-2+\frac{4a}{2+a}-\frac{a^3+b}{a^2+2a}$.

1) $\frac{a+1}{a^3-1}-\frac{1}{a^2+a+1}$; 3) $\frac{a+b}{a^2-ab+b^2}-\frac{1}{a+b}$;

2) $\frac{a^2+4}{a^3+8}-\frac{1}{a+2}$; 4) $\frac{m^2-3m+9}{m^3-27}-\frac{1}{m-3}$.

Ifodani soddalashtirib, so‘ngra son qiymatini toping:

1) $\frac{8a^2}{a^3-1}+\frac{a+1}{a^2+a+1}$, bunda $a=2$;

2) $\frac{3c^2-c+3}{c^3-1}-\frac{c-1}{c^2+c+1}+\frac{2}{1-c}$, bunda $c=1\frac{1}{2}$.

27-§. Algebraik kasrlarni ko'paytirish va bo'lish

Algebraik kasrlarni ko'paytirish va bo'lish ham oddiy larni ko'paytirish va bo'lish qoidalari bo'yicha bajariladi:

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd};$$

$$\frac{a}{b}:\frac{c}{d}=\frac{ad}{bc}.$$

1- masala. Kasrlarni ko'paytiring:

$$\frac{1}{2xy},\ \frac{4x^2y^3}{5z}\ \text{va}\ \frac{10z^2}{3x^3}.$$

△ $\frac{1}{2xy}\cdot\frac{4x^2y^3}{5z}\cdot\frac{10z^2}{3x^3}=\frac{1\cdot4x^2y^3\cdot10z^2}{2xy\cdot5z\cdot3x^3}=\frac{4y^2z}{3x^2}$. ▲

2- masala. $\frac{a-b}{a^2+ab}$ va $\frac{b^2+ab}{(a-b)^2}$ kasrlarni ko'paytiring.

△ Ko'paytuvchilarga ajratib, topamiz:

$$\frac{a-b}{a^2+ab}\cdot\frac{b^2+ab}{(a-b)^2}=\frac{(a-b)\,b\,(a+b)}{a\,(a+b)\,(a-b)^2}=\frac{b}{a(a-b)}.$$ ▲

3- masala. $\frac{m+n}{9m^2n^3}$ va $\frac{m^2-n^2}{27mn^2}$ kasrlarni bo'ling.

△ $\frac{m+n}{9m^2n^3}:\frac{m^2-n^2}{27mn^2}=\frac{(m+n)\cdot27mn^2}{9m^2n^3\,(m^2-n^2)}=\frac{(m+n)\,3}{mn\,(m-n)\,(m+n)}=\frac{3}{mn\,(m-n}$

Algebraik kasrni darajaga ko'tarishda ushbu $\left(\frac{a}{b}\right)^n$ formuladan foydalanilishini eslatib o'tamiz.

Masalan,

$$\left(\frac{4a^2}{b}\right)^2=\frac{16a^4}{b^2};\ \left(\frac{a+b}{3c}\right)^3=\frac{(a+b)^3}{27c^3}.$$

Kasrlarni ko‘paytiring **(505—506):**

1) $\frac{85}{24}\cdot\frac{72}{17}$; 2) $\frac{256}{169}\cdot\frac{13}{64}$; 3) $50\cdot\frac{7}{625}$; 4) $\frac{5}{26}\cdot 39$.

1) $\frac{a^3b}{c}\cdot\frac{c^2}{a^4}$; 3) $\frac{6a}{5b}\cdot\frac{15c}{2d}$; 5) $\frac{2a}{3b}\cdot 3c$;

2) $\frac{m^2n^2}{k}\cdot\frac{k^3}{m^3n^3}$; 4) $\frac{4m}{9n}\cdot\frac{27k}{16d}$; 6) $14a^2\cdot\frac{b^2}{7c^3}$.

Kasrlarni bo‘ling:

1) $\frac{3}{5}:\frac{3}{7}$; 3) $\frac{a}{8}:\frac{1}{3}$; 5) $\frac{2}{a}:\frac{6}{7}$;

2) $\frac{11}{12}:\frac{2}{5}$; 4) $\frac{6}{c}:\frac{m}{13}$; 6) $\frac{9}{35}:\frac{b}{5}$.

Kasrlarni bo‘ling:

1) $\frac{8}{17}:\frac{8}{17}$; 3) $\frac{3a}{7b}:\frac{a}{b}$; 5) $\frac{2a}{3b}:\frac{a^2}{bc}$;

2) $\frac{a}{b}:\frac{a}{b}$; 4) $\frac{c}{2d}:\frac{4c^2}{5d}$; 6) $\frac{5m}{n^2}:\frac{10m^3}{n}$.

Kasrlarni bo‘ling:

1) $\frac{17}{12}:\frac{34}{39}$; 3) $\frac{4}{13}:5$; 5) $12:\frac{8}{9}$;

2) $\frac{54}{25}:\frac{81}{75}$; 4) $\frac{a}{b}:c$; 6) $a:\frac{b}{c}$.

Kasrlarni bo‘ling:

1) $\frac{a^2b}{c}:\frac{a^4}{c^2}$; 3) $\frac{4a}{5b}:\frac{12c}{25d}$; 5) $\frac{6a}{5b}:(5c)$;

2) $\frac{mn}{k}:\frac{m^2n^2}{k^3}$; 4) $\frac{8m}{9n}:\frac{16k}{27d}$; 6) $12a^2:\frac{4d}{5c^2}$.

Ko‘rsatilgan amallarni bajaring **(511—517):**

1) $\left(\frac{5a}{7b}\right)^2\cdot\frac{14b^2}{25a^3}$; 2) $\left(\frac{3a^2}{2b}\right)^3\cdot\frac{16b^3}{21a^4}$; 3) $\frac{2a^2}{5b^2}:\frac{12a^2}{15b^2}$;

512. 1) $\frac{8a^2b}{9c} \cdot \frac{36c^3}{5a^3b}$;

2) $\frac{7b^4}{9c^5y} : \frac{35b^4c^2}{18c^4y^2}$;

3) $\frac{16x^2y}{7z} : \frac{20xy^3}{21z^2}$;

4) $\frac{46d^3c}{15a} : \frac{23dc^2}{5a^3}$;

5) $\frac{18m^3n^5}{7k} : (9n$

6) $24k^2 : \frac{12m^4}{11p^3}$

513. 1) $\frac{3x^2y}{4a^2b} \cdot 4a^2b$;

2) $\frac{5a^2b}{7xy^2} \cdot 14xy^2$;

3) $15xy : \frac{30xy}{7a^2b}$;

4) $\frac{7x^2y}{2a^2b} : (14x^2y)$.

514. 1) $\frac{7-x}{a+b} \cdot \frac{a-b}{7-x}$;

2) $\frac{x-y}{2a} \cdot \frac{4b}{x-y}$;

3) $\frac{c+d}{c-d} : \frac{c}{c-d}$;

4) $\frac{a-b}{2b} : \frac{a-b}{6b^2}$;

5) $\frac{a^2-ab}{b}$ ·

6) $\frac{ab+b^2}{9}$:

515. 1) $\frac{a+1}{b} \cdot \frac{4b^2}{a^2-1}$;

2) $\frac{1-a}{3b^2} \cdot \frac{b^3}{1-a^2}$;

3) $\frac{a^2-b^2}{9b^2} : \frac{a+b}{3b}$;

4) $\frac{5m}{m^2-n^2} : \frac{15m^3}{m-n}$;

5) $\frac{3(x+y)}{4y^2(x^2+y^2)} \cdot \frac{x^2+y^2}{x^2-y^2}$;

6) $\frac{5(a-b)}{3(a^2+b^2)} : \frac{(a-b)^2}{a^2+b^2}$.

516. 1) $\frac{a^2-b^2}{3a+3b} \cdot \frac{3a^2}{5b-5a}$;

2) $\frac{5x^2-5y^2}{x^2+y^2} \cdot \frac{3x^2}{10y-10x}$;

3) $\frac{a^2-25}{a^2-3a} : \frac{a+5}{9-a^2}$;

4) $\frac{3n^2-3m^2}{n^2+np} \cdot \frac{6m-6n}{n+p}$;

5) $\frac{a^2+b^2}{x^3+x^2y} \cdot \frac{x^2-y^2}{a^4-b^4}$;

6) $\frac{a^2+b^2}{a^2-ab} : \frac{a^4b-b^5}{a^2b-ab^2}$.

$a^2+6a+9 \quad a^2-25$ $\qquad a^2+2ab+b^2 \quad a-7$

2) $\dfrac{b^2-8b+16}{b+3} : \dfrac{(b-4)^2}{b^2-9}$;

4) $\dfrac{a^2-2a+1}{2a+1} : \dfrac{a-1}{4a^2-1}$.

-§ Algebraik kasrlar ustida birgalikda bajariladigan amallar

Algebraik kasrlar ustida birgalikda bajariladigan amallarga misollar ko‘ramiz.

- masala. Ifodani soddalashtiring: $\left(\dfrac{a+1}{2a-2}-\dfrac{1}{2a^2-2}\right)\cdot\dfrac{2a+2}{a+2}$.

Qavs ichidagi ifodalarni soddalashtiraylik:

$$\frac{a+1}{2a-2}-\frac{1}{2a^2-2}=\frac{a+1}{2(a-1)}-\frac{1}{2(a^2-1)}=\frac{(a+1)^2-1}{2(a^2-1)}=$$

$$=\frac{(a+1-1)(a+1+1)}{2(a^2-1)}=\frac{a(a+2)}{2(a+1)(a-1)}.$$

baytmani topamiz:

$$\frac{a(a+2)}{2(a+1)(a-1)}\cdot\frac{2a+2}{a+2}=\frac{a(a+2)2(a+1)}{2(a+1)(a-1)(a+2)}=\frac{a}{a-1}. \blacktriangle$$

- masala. Ko‘rsatilgan amallarni bajaring:

$$\left(\frac{a+b}{a-b}-\frac{a-b}{a+b}\right):\left(\frac{a+b}{a-b}-1\right).$$

Birinchi qavs ichidagi amalni bajaramiz:

$$\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{(a+b)^2-(a-b)^2}{(a-b)(a+b)}=\frac{(a+b+a-b)(a+b-a+b)}{a^2-b^2}=$$

$$=\frac{2a\cdot 2b}{a^2-b^2}=\frac{4ab}{a^2-b^2}.$$

Ikkinchi qavs ichidagi amalni bajaramiz:

$$\frac{a+b}{a-b}-1=\frac{a+b-a+b}{a-b}=\frac{2b}{a-b}.$$

Bo‘lamiz:

$$\frac{4ab}{a^2-b^2}:\frac{2b}{a-b}=\frac{4ab\left(a-b\right)}{\left(a^2-b^2\right)2b}=\frac{2a}{a+b}.\blacktriangle$$

3- masala. Hovuz birinchi quvur orqali a soatda, ikkinchisi orqali b soatda to‘ladi. Agar bir vaqtda ikkala quvurni ochib qo‘yilsa, hovuz necha soatda to‘ladi?

△ Hovuzning hajmi V bo‘lsin, deylik. Bir soatda birinchi quvur $\frac{V}{a}$ ga teng hajmni, ikkinchisi $\frac{V}{b}$ ga teng hajmni to‘ldiradi, ikkala quvur esa bir soatda $\frac{V}{a}+\frac{V}{b}$ ga teng hajmni to‘ldiradi. Qidirilayotgan vaqt t bo‘lsin. t soatda ikkala quvur hovuzni butunlay to‘ldirishi kerak, ya’ni

$$\left(\frac{V}{a}+\frac{V}{b}\right)t=V.$$

Tenglikning ikkala qismini V ga bo‘lib,

$$\left(\frac{1}{a}+\frac{1}{b}\right)t=1$$

ni hosil qilamiz. Qavs ichida turgan kasrlarning yig‘indisi $\frac{a+b}{ab}$ ga teng. Shuning uchun $\frac{a+b}{ab}t=1$, bundan $t=\frac{ab}{a+b}$. ▲

Mashqlar

Ko‘rsatilgan amallarni bajaring **(518—523):**

518. 1) $\left(\frac{a}{2}-\frac{a}{3}\right)\cdot\frac{1}{a^2}$; 3) $\frac{a-b}{a+b}\left(\frac{a}{5}+\frac{b}{5}\right)$; 5) $1:\left(1+\frac{1}{a}\right)$;

2) $\frac{a^2}{3}\cdot\left(\frac{2}{a}+\frac{2}{a^2}\right)$; 4) $\frac{ab}{a-b}\left(\frac{1}{b}-\frac{1}{a}\right)$; 6) $b:\left(b+\frac{1}{b}\right)$.

519. 1) $\left(1+\frac{1}{a}\right):\left(1-\frac{1}{a}\right)$; 3) $\left(\frac{b}{a}+\frac{a}{b}-2\right):\left(\frac{1}{b}-\frac{1}{a}\right)$;

2) $\left(a+\frac{a}{b}\right)\left(a-\frac{a}{b}\right)$; 4) $\left(\frac{m}{n}+\frac{n}{m}+2\right)\left(1+\frac{m-n}{m+n}\right)$.

520. 1) $\left(1-\frac{a-b}{a+b}\right)\left(2+\frac{2b}{a-b}\right)$; 3) $\left(\frac{6}{a-b}-\frac{5}{a+b}\right)\cdot\frac{a-b}{a+11b}$;

2) $\left(1+\frac{a+b}{a-b}\right)\left(2-\frac{2a}{a+b}\right)$; 4) $\left(\frac{3}{c}+\frac{3}{c+d}\right)\cdot\frac{c}{18(2c+d)}$.

521. 1) $\left(\frac{2m+1}{2m-1}-\frac{2m-1}{2m+1}\right):\frac{4m}{10m-5}$; 3) $\frac{y-1}{y}:\left(\frac{y^2+1}{y^2+2y}-\frac{2}{y+2}\right)$;

2) $\left(\frac{z+6}{3z+9}-\frac{1}{z+3}\right):\frac{z+2}{27z}$; 4) $\frac{m-2}{m-5}:\left(\frac{m^2+24}{m^2-25}-\frac{4}{m-5}\right)$.

522. 1) $\frac{a^2+ab}{a^2+b^2}\cdot\left(\frac{a}{a-b}-\frac{b}{a+b}\right)$; 3) $\left(\frac{c+d}{c}-\frac{2c}{c-d}\right)\cdot\frac{d-c}{c^2+d^2}$;

2) $\frac{ab-b^2}{a^2+b^2}\cdot\left(\frac{a}{a+b}+\frac{b}{a-b}\right)$; 4) $\left(\frac{2c}{c+d}+\frac{d-c}{c}\right)\cdot\frac{c+d}{c^2+d^2}$.

523. 1) $\left(\frac{a+1}{2a-2}+\frac{6}{2a^2-2}-\frac{a+3}{2a+2}\right)\cdot\frac{4a^2-4}{3}$;

2) $\left(\frac{b}{a^2+ab}+\frac{2}{a+b}+\frac{a}{b^2+ab}\right):\frac{a^2-b^2}{4ab}$;

3) $\frac{a^2-c^2}{a+b}\cdot\frac{a^2-b^2}{ac+c^2}\cdot\left(a+\frac{ac}{a-c}\right)$;

4) $\frac{c^2-ac}{a^2-b^2}\cdot\frac{a-b}{c^2-a^2}:\left(c-\frac{ac}{a+c}\right)$.

524. Hajmi V bo‘lgan muz bo‘lagining massasi p kilogrammga teng. Hajmi V_1 bo‘lgan bo‘lakning massasi nimaga teng?

525. Avtomobil soatiga v kilometr tezlik bilan harakat qilib, s kilometr yo‘l bosib o‘tdi. Agar mototsiklchining tezligi soatiga u kilometr bo‘lsa, shu vaqt ichida u qancha yo‘l bosib o‘tadi?

526. Motorli qayiqning turg‘un suvdagi tezligi soatiga v kilometr, daryo oqimining tezligi esa v_1 kilometr. Qayiq oqim bo‘yicha harakat qilib, s kilometr o‘tdi. Motorli qayiq oqimga qarshi shu vaqt ichida qancha masofani bosib o‘tadi?

527. *(Abu Rayhon Beruniy masalasi.)* Ikki buyumdan birining 10 tasi bir dinor (pul birligi) va ikkinchisining 15 tasi bir dinor. Bir dinorga ikkala buyumdan bir xil miqdorda necha donadan sotib olish mumkin?

O‘zingizni tekshirib ko‘ring!

1. Harflarning kasr ma’noga ega bo‘ladigan qiymatlarini toping:

$$\frac{a}{b};\ \frac{3}{c-1};\ \frac{k}{d+2}.$$

2. Amallarni bajaring:

1) $4a+\dfrac{1-4a^2}{a}$;

2) $\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}$;

3) $\dfrac{2a-4}{3b}\cdot\dfrac{6b}{a-2}$;

4) $\dfrac{a^2-b^2}{b^2}:\dfrac{a+b}{b}$.

3. Ifodani soddalashtiring va uning $x=2\dfrac{2}{3}$ bo‘lgandagi son qiymatini toping:

$$\frac{1+2x}{x-3}-\frac{x^2+3x}{5}\cdot\frac{10}{x^2-9}.$$

V bobga doir mashqlar

Kasrlarni umumiy maxrajga keltiring:

528. 1) $\dfrac{5a}{a^3-27}$, $\dfrac{a-3}{a^2+3a+9}$ va $\dfrac{1}{a-3}$; | 2) $\dfrac{3}{x+2}$, $\dfrac{x+1}{x^3+8}$ va $\dfrac{x+2}{x^2-2x+4}$.

Amallarni bajaring **(529—530):**

529. 1) $\frac{a+3}{5}+\frac{7+a}{10}+\frac{a-3}{2}$;

2) $\frac{b-7}{4}+\frac{5b-2}{3}+\frac{3b-1}{8}$;

3) $\frac{a-2}{45}-\frac{a+5}{15}-\frac{a-9}{9}$;

4) $\frac{b}{12}-\frac{3b+1}{9}-\frac{2b-1}{4}$.

530. 1) $\frac{y}{n-2}+\frac{z}{2-n}$;

2) $\frac{p+2q}{3p-q}-\frac{5q-2p}{q-3p}$;

3) $\frac{2m}{3-5n}-1+\frac{7n-4}{5n-3}$;

4) $4-\frac{3a}{5-2b}+\frac{5(a-10)}{2b-5}$.

Ko‘rsatilgan amallarni bajaring **(531—533):**

531. 1) $\frac{a^2-2ab+b^2}{a^2-ab+b^2}:\frac{8a-8b}{a^3+b^3}$;

2) $\frac{a^2+2ab+b^2}{a^2+ab+b^2}\cdot\frac{a^3-b^3}{7a+7b}$;

532. 1) $\frac{64x^2-1}{x^2-4}\cdot\frac{(x+2)^2}{x^2-4}\cdot\frac{(x-2)^2}{8x+1}$;

2) $\frac{x-6}{x^2+6x+9}\cdot\frac{x^2+4x+4}{(x^2+2)(x-2)}\cdot\frac{x^3-9x}{(x-6)(x+2)}$;

3) $\frac{am^2-an^2}{m^2+2mn+n^2}:\frac{am^2+2amn+an^2}{3m+3n}$;

4) $\frac{ab-4b-2a+8}{2a+8-ab-4b}:\frac{2a-8-ab+4b}{ab+4b-2a-8}$.

533. 1) $(x^2-1)\left(\frac{1}{x-1}-\frac{1}{1+x}+1\right)$;

2) $\left(1+a-\frac{a^2+3}{a+1}\right)(1-a^2)$;

3) $\left(\frac{x+y}{x-y}-\frac{x-y}{x+y}\right):\left(\frac{x-y}{x+y}+\frac{x+y}{x-y}\right)$;

4) $\left(\frac{2-a}{2+a}-\frac{a+2}{a-2}\right):\left(\frac{2+a}{2-a}+\frac{a-2}{a+2}\right)$.

№ 10 *n sonning raqamlari yig‘indisi 2006 ga teng. n sonni ikkita o‘zaro teng sonlar ko‘paytmasi ko‘rinishida tasvirlash mumkinmi?*

V bobga doir sinov mashqlari — testlar

1. Kasrni qisqartiring: $\frac{27a^2-36ab+12b^2}{9a^2-4b^2}$.

A) $\frac{3(3a-2b)}{3a+2b}$; B) $\frac{3a-2b}{3a+2b}$;

C) $\frac{39-36ab}{5}$; D) $\frac{3a^2-36ab+3b^2}{a^2-b^2}$.

2. Kasrni qisqartiring: $\frac{7a^2(ab^2-9a)}{3a(21a-7ab)}$.

A) $\frac{7a(ab^2-9a)}{3(21a-7ab)}$; B) $\frac{-a(b+3)}{3}$;

C) $\frac{7(ab^2-9a)}{3(21-7b)}$; D) $\frac{a(b-3)}{3}$.

3. Amallarni bajaring: $\frac{4}{a+b}+\frac{5}{a-b}-\frac{10b}{a^2-b^2}$.

A) $\frac{9}{a-b}$; B) $\frac{9}{a+b}$; C) $\frac{-9}{a+b}$; D) $\frac{9(a+b)}{a-b}$.

4. Kasrlarni ayiring: $\frac{a^2+9}{a^3+27}-\frac{1}{a+3}$.

A) $\frac{1}{a^2+9}$; B) $\frac{3}{a^2+9}$; C) $\frac{a}{a^3+9}$; D) $\frac{3a}{a^3+27}$.

5. Kasrlarni ko‘paytiring: $\frac{9a^2-16b^2}{6a+8b}\cdot\frac{6a^2}{12b-9a}$.

A) a^2; B) $-a^2$; C) $\frac{a^2}{3a-4b}$; D) $\frac{6}{3a+4b}$.

6. Kasrlarni bo‘ling: $\frac{4a^2-20ab+25b^2}{5b+4}:\frac{(2a-5b)^2}{25b^2-16}$.

A) $\frac{5b+4}{2a-5b}$; B) $\frac{2a-5b}{5b-4}$; C) $5b-4$; D) $5b+4$.

7. Kasrni qisqartiring: $\dfrac{8a^2-22ab+15b^2}{16a^2-25b^2}$.

A) $\dfrac{2a-3b}{4a+5b}$; B) $\dfrac{2a+3b}{4a-5b}$; C) $\dfrac{4a-5b}{4a+5b}$; D) $\dfrac{4a+3b}{2a-5b}$.

8. Kasrlarni ayiring: $\dfrac{9x^2+16}{27x^3+64}-\dfrac{1}{3x+4}$.

A) $\dfrac{9x^2+16}{3x+4}$; B) $\dfrac{-12x}{27x^3+64}$; C) $\dfrac{12x}{27x^3+64}$; D) $\dfrac{9x^2+4}{27x^3-64}$.

9. Amallarni bajaring: $\dfrac{4}{3a+2b}-\dfrac{2}{2b-3a}+\dfrac{8b}{4b^2-9a^2}$.

A) $\dfrac{6}{3a-2b}$; B) $\dfrac{6}{3a+2b}$; C) $\dfrac{12a}{9a^2-4b^2}$; D) $\dfrac{12b}{2b-3a}$.

Tarixiy ma'lumotlar

Qisqa ko'paytirish formulalari, algebraik kasrlarga oid ma'lumot qadimgi risolalarda uchraydi. Masalan, al-Karajining „Al-Fahri", Misr olimi Abu Komil (850—930) ning „Kitab al-jabr val-muqobala" asarlarida ham algebraik kasrlar o'rganilgan. Abu Komil al-Xorazmiydan keyin algebraga doir kitob yozgan birinchi olimdir. Abu Komil o'z asarida

$$\left(\frac{a}{b}\right)\cdot b=a,\quad \frac{a}{b}=\frac{a^2}{ab},\quad \frac{a}{b}\cdot\frac{b}{a}=1,\quad \frac{a}{b}+\frac{b}{a}=\frac{a^2+b^2}{ab}$$

kabi sodda munosabatlarga ham e'tibor qaratadi.

Algebraik kasrlarga I. Nyutonning „Umumiy arifmetika" kitobida ham yetarlicha o'rin berilgan. „$\frac{a}{b}$ kasr a ni b ga bo'lish natijasida hosil bo'lgan kattalikdir. Xuddi shuningdek, $\frac{ab-bb}{a+x}$ kattalik $ab-bb$ ni $a+x$ ga bo'lish natijasida hosil bo'ladi," — deydi Nyuton.

Sizlar bilan buyuk vatandoshimiz al-Xorazmiy asos solgan algebra fanining boshlang'ich tushunchalari va natijalari bilan tanishdik.

VI BOB
KOMBINATORIKA ELEMENTLARI

29-§ Kombinatorikaning asosiy qoidasi

Aziz o‘quvchi! Siz 6- sinfda kombinatorikaning qo‘shish va ko‘paytirish qoidalariga oid dastlabki tushunchalar bilan tanishgansiz.

1- masala. Samarqanddan Toshkentga 4 xil yo‘l bilan kelish mumkin: samolyot, poyezd, avtobus va yengil mashina (taksi). Toshkentdan Xo‘jakentga 3 xil transport vositasi olib boradi: poyezd, avtobus, taksi. Samarqanddan Xo‘jakentga necha xil usulda kelish mumkin (22- rasm)?

22- rasm.

△ Samarqanddan Toshkentga kelishning jami 4 ta yo‘li bor. Mavjud 4 ta yo‘ldan bittasini tanlab, Toshkentga keldik, deylik. Endi Xo‘jakentga borishning 3 ta yo‘li — imkoniyati bor. Shunday qilib, Samarqanddan Toshkent orqali Xo‘jakentga borishning jami $4 \cdot 3 = 12$ xil usuli bor.

Javob: 12 xil. ▲

> Umuman, A shahardan B shaharga kelishning m ta, B dan C shaharga kelishning n ta yo‘li bo‘lsa, u holda A dan C ga kelishning jami $m \cdot n$ ta yo‘li bor, ya’ni A dan C ga $m \cdot n$ xil usuli bilan kelish mumkin.

Bu qoida ko‘paytirish qoidasidir va u kombinatorikaning asosiy qoidasi hisoblanadi.

2- masala. „Makro“ supermarketining „Hammasi uy uchun“ bo‘limida 5 xil piyola, 6 xil taqsimcha, 4 xil choy

qoshiq bor. Nargiza xola turli nomdagi ikkita buyum sotib olmoqchi. U buni necha xil usulda amalga oshirishi mumkin?

△ 1) Piyola va taqsimchani $5 \cdot 6 = 30$ usulda; 2) Piyola va qoshiqni $5 \cdot 4 = 20$ usulda; 3) taqsimcha va qoshiqni $6 \cdot 4 = 24$ xil usulda olish mumkin. Demak, turli nomdagi ikkita buyumni $30 + 20 + 24 = 74$ xil usulda tanlab olish mumkin ekan.

Javob: 74 xil usulda.▲

3- masala. Nechta uch xonali sonda faqatgina bitta 7 raqami bor?

△ 7 raqami 1-, 2-, 3- o‘rinda (yuzlar, o‘nlar, birlar xonasida) bo‘lishi mumkin.

Agar 7 raqami 1- o‘rinda turgan bo‘lsa, 2- va 3- o‘rinlarni $9 \cdot 9 = 81$ usulda to‘ldirish mumkin.

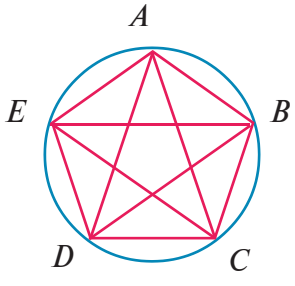
Agar 7 raqami 2- o‘rinda bo‘lsa, u holda 1- o‘rinda 0 va 7 raqamlaridan boshqa ixtiyoriy raqam turishi mumkin. 1- o‘rinni egallashning $10 - 2 = 8$ ta imkoniyati bor. Bu holda 3- o‘rinda 7 raqamidan boshqa ixtiyoriy raqam tura oladi; demak, imkoniyatlar soni $8 \cdot 9 = 72$ ta.

Agar 7 raqami 3- o‘rinda tursa, u holda 1- o‘rinni olish uchun 8 ta, 2- o‘rinni olish uchun esa 9 ta imkoniyat bor. Shunday qilib, o‘nli yozuvida faqatgina bitta 7 raqami bor uch xonali sonlar jami $81 + 72 + 72 = 225$ ta ekan.

Javob: 225 ta.▲

4- masala. Aylanada olingan 5 ta nuqta *A, B, C, D, E* harflari bilan belgilangan. Har bir nuqta qolgan har bir nuqta bilan tutashtirilsa, nechta kesma hosil bo‘ladi (23- rasm)?

△ ***1- usul.*** Nuqtalar soni kam bo‘lgani uchun, masalaga mos shaklni chizib, kesmalar sonini bevosita sanab chiqish mumkin, ular — 10 ta. Ammo aylanada olingan nuqtalar soni ko‘p bo‘lsa (masalan, 100 ta, ...), mos shakl chizish va undagi kesmalarni bevosita sanash qiyinlashadi. Bu holda boshqa yo‘l tutish kerak.



23- rasm.

2- usul. Aylanada olingan 5 ta nuqtaning har biridan 4 tadan kesma o‘tkaziladi. Bunday kesmalar soni $5 \cdot 4 = 20$ ta, ammo kesmalar sonini hisoblashda har bir kesma ikki marta sanalgan. Demak, biz 20 ni 2 ga bo‘lishimiz kerak: $20 : 2 = 10$.

3- usul. A nuqtani qolgan 4 ta nuqta bilan tutashtirsak, 4 ta kesma hosil qilamiz: AB, AC, AD, AE. B nuqtadan ham 4 ta kesma o‘tkazish mumkin, ammo B dan o‘tkazilgan bitta kesma ($BA = AB$) ni biz sanadik. Demak, B nuqtadan 3 ta yangi (avval hisoblanmagan, sanalmagan) kesma o‘tkaziladi. Shunga o‘xshash, C dan 2 ta, D dan esa 1 ta yangi kesma o‘tkazish mumkin. E nuqtadan o‘tkaziladigan 4 ta kesmaning hammasi avval hisoblangan ($EA = AE$; $EB = BE$; $EC = CE$; $ED = DE$). Demak, aylanada belgilangan 5 ta nuqtani tutashtiruvchi jami kesmalar soni $4 + 3 + 2 + 1 + 0 = 10$ ta.▲

5- masala. 3, 4, 5, 6, 8, 9 raqamlari yordamida hammasi bo‘lib: 1) raqamlar takrorlanmasa; 2) raqamlar takrorlanishi mumkin bo‘lsa, nechta uch xonali son tuzish mumkin?

△ 1) Berilgan raqamlar 6 ta. Ularning xohlagan bittasi 3 xonali sonning birinchi raqami bo‘lishi mumkin. Demak, 3 xonali sonning birinchi raqamini tanlash imkoniyati 6 ta bo‘ladi. U holda 2- raqam qolgan 5 ta raqamning ixtiyoriy bittasi bo‘lishi mumkin, ya’ni 2- raqamni tanlash imkoniyatlarimiz 5 ta. Shunga o‘xshash, 3- raqamni tanlash imkoniyatlarimiz 4 ta.

Demak, raqamlar takrorlanmasa, jami uch xonali sonlar soni $6 \cdot 5 \cdot 4 = 120$ ta bo‘lar ekan.

Javob: 120 ta. ▲

△ 2) Raqamlar takrorlanadigan bo‘lsa, uch xonali sonning 1-, 2-, 3- xonalariga yoziladigan raqamni tanlash imkoniyatlari 6 tadan bo‘ladi, chunki berilgan raqamlar soni 6 ta. Bu holda jami 3 xonali sonlar soni $6 \cdot 6 \cdot 6 = 6^3 = 216$ ta bo‘ladi.

Javob: 216 ta. ▲

534. Onasi Nargizaga „Korzinka. Uz“ supermarketidan 3 xil meva xarid qilishni aytdi. „Korzinka. Uz“ da 6 xil olma, 4 xil nok, 5 xil uzum bor. Nargiza mevalarning har bir xilidan 1 kg dan olib, nechta turli to‘plam tuza oladi?

535. Nechta 4 xonali sonda faqatgina bitta 5 raqami bor?

536. Aylanada: a) 10 ta; b) 100 ta; d) n ta nuqta belgilangan. Har bir nuqta qolgan har bir nuqta bilan tutashtirilsa, har bir holda jami nechta kesma hosil bo‘ladi?

537. 1) 3; 2) 4; 3) 5; 4) 6; 5) 8; 6) 15 nafar do‘stlar o‘zaro qo‘l berib ko‘rishishdi. Har bir holda qo‘l berishlar soni nechta bo‘lgan?

538. 10 nafar o‘rtoq o‘zaro shaxmat turniri o‘tkazishmoqchi. Bunda har bir bola qolgan har bir bola bilan bir partiya shaxmat o‘ynaydi. Bu turnirda jami nechta partiya o‘ynaladi?

Ayting-chi, ***536 – 538-*** *masalalarning o‘xshashligi nimada?*

539. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 raqamlari yordamida hammasi bo‘lib: 1) raqamlar takrorlanmasa; 2) raqamlar takrorlanishi mumkin bo‘lsa, nechta uch xonali son tuzish mumkin?

540. 1, 2, 3, 4, 5 raqamlari yordamida nechta: a) ikki xonali; b) uch xonali; d) to‘rt xonali sonlar yozish mumkin? Raqamlar: takrorlanmaydigan; takrorlanadigan hollarni alohida ko‘ring.

541. Futbol bo‘yicha jahon chempionatida oltin, kumush, bronza medallari uchun bo‘ladigan o‘yinlarda 16 ta jamoa qatnashmoqda. Medallar jamoalar orasida necha xil usul bilan taqsimlanishi mumkin?

542. Bir mamlakatda 4 ta shahar bor ekan: A, B, C va D. A shahardan B ga 6 ta yo‘l, B shahardan C ga 4 ta yo‘l olib borarkan. A dan D ga 2 ta yo‘l, D dan C ga 3 ta yo‘l bilan

borish mumkin ekan. A shahardan C shaharga necha xil yo‘l bilan borish mumkin?

543. Agar natural sonning yozuvida faqat toq sonlar qatnashsa, bunday sonni „yoqimtoy“ son deymiz. Nechta: 1) 3 xonali; 2) 4 xonali „yoqimtoy“ son mavjud?

544. Yozuvida hech bo‘lmaganda bitta juft raqam qatnashgan 6 xonali sonlar nechta?
Ko‘rsatma: Yozuvida faqat toq sonlar qatnashgan 6 xonali sonlar soni $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^6 = 15\,625$ ta. Jami 6 xonali sonlar esa 900 000 ta. Masala shartini qanoatlantiradigan 6 xonali sonlar soni $900\,000 - 15\,625 = 884\,375$ ta.

545. 4 ta turli xatni 4 ta turli konvertga necha xil usulda joylash mumkin?

546. 5 nafar o‘quvchidan 2 nafarini „Bilimlar bellashuvi“ da qatnashish uchun tanlab olish kerak. Buni necha xil usulda bajarish mumkin?

547. Doskada 12 ta ot, 8 ta fe’l va 7 ta sifat yozilgan. Gap tuzish uchun har bir so‘z turkumidan bittadan olish kerak. Buni necha xil usul bilan amalga oshirish mumkin?

24- rasm.

25- rasm.

548. 1) Shaxmat taxtasida oq va qora ruxni bir-birini ololmaydigan („ura olmaydigan“) qilib necha xil usulda joylashtirish mumkin (24- rasm)?
2) Shaxmat taxtasida 8 ta ruxni bir-birini ololmaydigan qilib necha xil usulda joylashtirish mumkin (25- rasm)?

549. Shaxmat taxtasiga oq va qora farzinlarni, ular bir-birini „ura olmaydigan“ qilib necha xil usulda joylashtirish mumkin?

550. Shaxmat taxtasiga oq va qora shohlarni, o‘yin qoidalarini buzmagan holda, necha xil usulda qo‘yish mumkin?
Ko‘rsatma: 3 ta holni qarang:
1) oq shoh burchakda turibdi;
2) oq shoh taxtaning chetida (lekin burchakda emas) turibdi;
3) oq shoh taxtaning chetida emas.

551. Maktab oshxonasida oq non, qora non va uch xil kolbasa bor. Ulardan necha xil buterbrod tayyorlash mumkin?

552. Ba’zi mamlakatlarning bayroqlari turli rangdagi 3 ta gorizontal yoki 3 ta vertikal „yo‘l“ lardan iborat. Oq, yashil, ko‘k rangli matolar yordamida shunday bayroqlardan necha xilini tikish mumkin?

553. Bo‘sh joylarga 1, 2, 3, 4, 5, 6, 7, 8 raqamlaridan birini yozish mumkin bo‘lsa, $\bigcirc + \square + \triangle = 10$ „tenglama“ nechta yechimga ega bo‘ladi? Raqamlar takrorlanishi mumkin. Ikki holni qarang (masalan: 1) 1, 1, 8; 1, 8, 1; 8, 1, 1 turli yechim; 2) bitta yechim deb qaraladigan hollar).

554. Nodirning chamadoni kod bilan ochiladi. Bu kod uchta raqamdan iborat bo‘lib, har bir raqam 3 dan katta emas. Kodda 13 soni qatnashmaydi. Nodir kodni unutib qo‘ygan bo‘lsa, kodni topish uchun u ko‘pi bilan necha marta „urinishi“ lozim bo‘ladi?

555. Ko‘p qavatli uyda yo‘lak eshigidagi qulf kod bilan ochiladi. Kod 0 va 1 raqamlaridan tuzilgan 4 xonali son (0000 va 1111 sonlar kod emas deb hisoblangan.) Qulf kodini unutgan bo‘lsangiz, eshikni eng ko‘pi bilan nechta urinishda ocha olasiz?
Ko‘rsatma: Avval bitta 1 qatnashgan kodlarni, keyin ikkita 1 bo‘lgan kodlarni va nihoyat, uchta 1 bo‘lgan kodlarni sinash kerak.

556. 20 kg guruchni 1 kg, 2 kg, 5 kg li toshlar yordamida pallali tarozida necha xil usulda tortish mumkin?

△ Bu ishni quyidagicha bajarish mumkin:
1) faqat 1 kg li tosh yordamida 1 ta usul;
2) faqat 2 kg li tosh yordamida 1 ta usul;
3) faqat 5 kg li tosh yordamida 1 ta usul;
4) 1 kg va 2 kg li toshlar yordamida 9 ta usul bilan:

1 kg li tosh	18	16	14	12	10	8	6	4	2
2 kg li tosh	1	2	3	4	5	6	7	8	9

5) 1 kg va 5 kg li toshlar yordamida 3 ta usul bilan:

1 kg li tosh	15	10	5
5 kg li tosh	1	2	3

6) 2 va 5 kg li tosh yordamida 1 ta usul: 5 ta 2 kg va 2 ta 5 kg;
7) 1 kg, 2 kg va 5 kg li toshlar yordamida 13 ta usul bilan:

	Usullar soni												
Toshlar, kg	1	2	3	4	5	6	7	8	9	10	11	12	13
1 kg	1	3	5	7	9	11	13	8	6	4	2	3	1
2 kg	7	6	5	4	3	2	1	1	2	3	4	1	2
5 kg	1	1	1	1	1	1	1	2	2	2	2	3	3

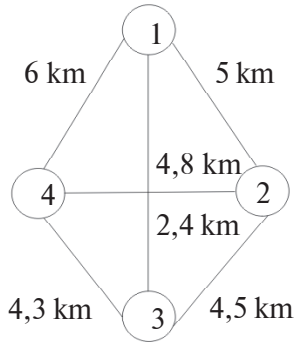
Demak, jami $1 + 1 + 1 + 9 + 3 + 1 + 13 = 29$ ta usul.
Javob: 29 ta usul.▲

557. 1) 1000 so‘mlik pulni 100, 200, 500 so‘mlik pullar bilan necha xil usulda maydalash mumkin?
2) 500 so‘mlik pulni 100 va 200 so‘mlik pullar bilan necha xil usulda maydalash mumkin?
3) 5000 so‘mlik pulni 100, 200, 500 va 1000 so‘mlik pullar yordamida necha xil usulda maydalash mumkin?

558. Firmaga 4 ta do‘kon tegishli. Inkassator (do‘kondagi pullarni yig‘ib bankka topshiruvchi xodim) 1-do‘kondan boshlab hamma do‘konlarni aylanib chiqadi va yana

1- do‘konga qaytib keladi. Mumkin bo‘lgan marshrutlardan eng qisqasini toping.

Ko‘rsatma: Har bir marshrut uchun 5 ta raqamli kod tuzing. Kodning birinchi va oxirgi raqami 1 bo‘lsin. Masalan, 12431 marshrutning uzunligi: $5 + 2{,}4 + 4{,}3 + 4{,}8 = 16{,}5$ (km).



559. Avtomashinalarni davlat ro‘yxatidan o‘tkazishda 3 ta raqam, 3 ta harfdan va shahar yoki viloyat uchun belgilangan koddan foydalaniladi. Masalan, avtomashina nomeridagi 01 kod — mashina Toshkentdan ro‘yxatga o‘tganini bildiradi. Nima deb o‘ylaysiz, Toshkentda eng ko‘pi bilan nechta avtomashina ro‘yxatdan o‘tishi mumkin?

△ Nomerlashda 24 ta harf qatnashadi, deylik. Nomer 6 ta „joy“ ni egallaydi. 1- „joy“ da 10 ta raqamdan ixtiyoriy biri bo‘lishi mumkin. 2- „joy“ ni 10 ta raqamdan biri egallaydi. 3- „joy“ da 9 ta raqamdan ixtiyoriy biri bo‘ladi. (3 ta bir xil raqamli nomer berilmaydi). Nomerdagi 1- harf ham, 2- harf ham, 3- harf ham 24 ta harfning ixtiyoriy biri bo‘lishi mumkin. Demak, Toshkentda ro‘yxatdan o‘tishi mumkin bo‘lgan jami avtomashinalar soni $10 \cdot 10 \cdot 9 \cdot 24 \cdot 24 \cdot 24 = 24^3 \cdot 900 = 12\ 441\ 600$ ta.

Bu hisoblashda harflarning nomerdagi 3 xonali sondan „bitta harf — 3 xonali son — 2 ta harf“ yoki „3 xonali son — 3 ta harf“ ko‘rinishida bo‘lishining farqi yo‘q.

Javob: 12 441 600 ta. ▲

30-§ O‘rin almashtirish. Guruhlash

1- masala. 4, 7, 9 raqamlaridan ularni takrorlamasdan nechta 3 xonali son tuzish mumkin?

Bu kabi masalalarni 6- sinfda ishlagansiz.

△ 1- o‘rinda berilgan 3 ta sondan ixtiyoriy bittasi turadi, ya’ni imkoniyatlar soni 3 ta. 2- o‘rinda qolgan 2 ta raqamdan

ixtiyoriy bittasi bo‘ladi, ya’ni 2- o‘rinni egallash imkoniyati 2 ta. Nihoyat, 3- o‘rinda qolgan bitta raqam turadi. Demak, shu 3 ta raqamdan tuzilishi mumkin bo‘lgan 3 xonali sonlar soni $3 \cdot 2 \cdot 1 = 3! = 6$ ta ekan. Shu 6 ta sonni yozaylik:

479, 497, 749, 794, 947, 974.

Hosil bo‘lgan 6 ta sonning tarkibi bir xil — ular berilgan 3 ta raqamdan tuzilgan, ammo ular bir-biridan raqamlarining *tartibi* bilan farqlanadi: 1, 2, 3 deb nomerlangan 3 ta o‘ringa 3 ta raqam turli tartibda joylashtirilgan. Bunday tartiblash (joylashtirish) *o‘rin almashtirish* deyiladi.

n ta: 1-, 2-, …, n- o‘ringa n ta a_1, a_2, … , a_n elementlarni bir o‘ringa bittadan qilib joylashtirish a_1, a_2, … , a_n elementlardan tuzilgan *o‘rin almashtirish* deyiladi.

n ta elementdan tuzilgan o‘rin almashtirishlar soni P_n bilan belgilanadi. Yuqoridagi misolda elementlar soni 3 ta edi, $n = 3$ va $P_3 = 3 \cdot 2 \cdot 1 = 3!$ ekanini ko‘rdik. Umuman, $P_n = n \cdot (n-1) \ldots 2 \cdot 1 = n!$

2- masala. 4 ta a, b, c, d elementdan (predmetdan) 2 tadan olib tuzilgan har xil guruhlar soni nechta?

△ 2 elementli guruhlarni tuzamiz:

$\{a, b\}$; $\{a, c\}$; $\{a, d\}$; $\{b, c\}$; $\{b, d\}$; $\{c, d\}$; — ularning soni 6 ta.

Javob: 6 ta. ▲

Umuman, n ta elementdan k tadan olib tuzilgan barcha guruhlar soni C_n^k deb belgilanadi va bu son $\frac{n!}{k!(n-k)!}$ ga teng:

$C_n^k = \frac{n!}{k!(n-k)!}$. C_n^k son n ta elementdan k tadan olib tuzilgan guruhlar soni deb o‘qiladi. Bizning misolda $n = 4$, $k = 2$ edi. Demak,

$$C_4^2 = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 2 \cdot 1 \cdot 2} = 6; \qquad C_n^k = \frac{n(n-1)\ldots(n-k+1)}{k!}$$

ekanini ko‘rsatish oson.

Haqiqatan ham,

$$C_n^k = \frac{1 \cdot 2 \cdot \ldots (n-k) \cdot (n-k+1)\ldots n}{k! 1 \cdot 2 \cdot \ldots (n-k)} = \frac{n(n-1)(n-2)\ldots(n-k+1)}{k!}.$$

Masalan, $C_5^2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{1\cdot 2\cdot 3\cdot 4\cdot 5}{1\cdot 2\cdot 1\cdot 2\cdot 3} = 10.$

Shu bilan birga, $C_5^2 = \frac{5\cdot 4}{2!} = 10.$

C_5^2 belgining yuqori indeksidagi 2 soni kasrning suratida 2 ta ko‘paytuvchi bo‘lishini bildiradi. Bu ko‘paytuvchilar: C_5^2 belgining quyi indeksidagi 5 va undan bitta kam bo‘lgan son 4 dir. Kasrning maxrajida esa yuqori indeksidagi son 2 gacha bo‘lgan natural sonlar ko‘paytmasi yoziladi: $2! = 1 \cdot 2$.

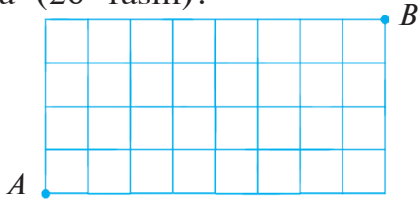
3- masala. Qavariq oltiburchakning diagonallari nechta nuqtada kesishadi? Hech qaysi uchta diagonal bitta nuqtada kesishmaydi, deb faraz qilinadi. Mos rasm chizing.

△ 2 ta diagonalning har bir kesishish nuqtasi oltiburchakning 4 ta uchini aniqlaydi. Oltiburchakning har 4 uchiga diagonallarning bitta kesishish nuqtasi mos keladi. Demak, kesishish nuqtalari soni 6 ta uchdan 4 ta uchni tanlashlar soniga teng ekan. Buni chizgan rasmingizdan sanab bilishingiz ham mumkin.

Javob: $C_6^4 = \frac{6\cdot 5\cdot 4\cdot 3}{1\cdot 2\cdot 3\cdot 4} = 15.$ ▲

C_n^k sonlarga *geometrik ma’no* berish mumkin.

4- masala. O‘lchamlari 7×4 bo‘lgan to‘g‘ri to‘rtburchak $7 \cdot 4 = 28$ ta kvadratchalarga bo‘lingan. Kvadratchalarning tomonlari bo‘yicha yurganda A dan B ga olib boruvchi eng qisqa yo‘llar soni nechta (26- rasm)?



26- rasm.

△ Kvadratchaning tomoni uzunligi 1 „qadam“ deyilsa, A dan B ga eng qisqa yo‘l bilan borish uchun 11 „qadam“ qo‘yishingiz shart, buning 7 „qadam“i gorizontal, 4 „qadam“i esa vertikal yo‘l bo‘yichadir. Shunday qilib, A dan B ga olib boruvchi eng qisqa yo‘llar soni jami 11 ta „qadam“dan 7 ta go-

rizontal „qadam“ ni tanlashlar soni C_{11}^{7} ga teng ekan. Ayni shu son 11 ta „qadam“ dan 4 ta vertikal „qadam“ni tanlashlar soniga ham tengdir, bundan $C_{11}^{7}=C_{11}^{4}$ ekani kelib chiqadi. Ammo $C_{11}^{4}=\frac{11\cdot10\cdot9\cdot8}{1\cdot2\cdot3\cdot4}=11\cdot10\cdot3=330.$

Javob: 330.▲

Agar to‘g‘ri to‘rtburchakning o‘lchamlari $m\times n$ bo‘lsa va u $m\cdot n$ ta kvadratchalarga ajratilgan bo‘lsa, u holda A dan B ga olib boruvchi eng qisqa yo‘llar soni $C_{m+n}^{n}=C_{m+n}^{m}$ bo‘ladi.

5- masala. 7 yigit va 4 qizdan iborat o‘quvchilar guruhidan oltita o‘quvchini shunday tanlab olish kerakki, ularning ichida qizlar soni ikkitadan kam bo‘lmasin. Buni necha xil usul bilan amalga oshirish mumkin?

△ Qizlarni guruhga 2, 3 va 4 ta tanlab olish mumkin. Ikkita qizni C_4^2 usul bilan, shundan so‘ng 4 ta yigitni C_7^4 usul bilan tanlab olamiz. Ko‘paytirish qoidasiga ko‘ra bunday usullar soni $C_4^2\cdot C_7^4$ ta. Agar avval uchta qiz tanlab olingan bo‘lsa, u holda $C_4^3\cdot C_7^3$ ta usul mavjud. Agar 4 ta qiz tanlab olingan bo‘lsa, $C_4^4\cdot C_7^2$ ta usul mavjud. Jami $C_4^2\cdot C_7^4+C_4^3\cdot C_7^3+C_4^4\cdot C_7^2=371$ ta usul bilan 6 kishidan iborat guruh tuzish mumkin.▲

6- masala. 1, 2, 3, ..., 9 raqamlaridan ularni takrorlamay tuzilgan 9 xonali sonlar ichida 2 va 5 raqamlari yonma-yon turadiganlari nechta?

△ Quyidagi hollar bo‘lishi mumkin: 2 birinchi o‘rinda, 5 ikkinchi o‘rinda, ..., 2 sakkizinchi o‘rinda, 5 to‘qqizinchi o‘rinda, bunday hollar soni 8 ta. Bundan tashqari, 2 va 5 larning yuqoridagi 8 holda o‘rinlarini almashtirib, yana 8 ta (ular yonma-yon turadigan) holni topamiz. Demak, 2 va 5 ni yonma-yon qilib, 16 usul bilan qo‘yish mumkin. Bu usullarning har biriga boshqa qolgan raqamlarning 7! ta o‘rin almashtirishlari mos keladi. Shunday qilib, 2 va 5 raqamlari yonma-yon turadigan o‘rin almashtirishlar soni $2\cdot8\cdot7!=2\cdot8!$ ga teng. ▲

560. P_4, P_5, P_6 sonlarini toping. Ularga qanday ma'no berish mumkin?

561. 2, 4, 7, 9 raqamlaridan ularni takrorlamasdan nechta 4 xonali son tuzish mumkin? Ularning nechtasi: 2 ga, 4 ga, 11 ga bo'linadi?

562. Tug'ilgan kuningizga taklif etilgan 4 ta do'stingizni 4 ta stulga necha xil usulda o'tkaza olasiz?

563. 1) C_{10}^4; 2) C_8^3; 3) C_7^5; 4) C_5^3 sonlarni ikki usulda hisoblang.

564. 1) $C_{10}^7 = C_{10}^3$; 2) $C_8^3 = C_8^5$; 3) $C_6^2 = C_6^4$ tengliklarning to'g'riligini bevosita hisoblab ko'rsating.

565. Kutubxonachi Sizga 5 ta turli kitobni o'qishni taklif qildi. Siz shulardan 3 tasini tanlab olmoqchisiz. Buni necha xil usulda amalga oshirish mumkin?

566. Ikkita parallel to'g'ri chiziq berilgan bo'lib, ularning bittasida 5 ta, ikkinchisida 3 ta nuqta belgilangan. Uchlari shu nuqtalarda bo'lgan nechta uchburchak mavjud?

567.

27- rasm.

A dan B ga olib boruvchi eng qisqa yo'llarni har bir shakl uchun alohida-alohida chizing (27- rasm).

568. Taqsimchada 8 ta yong'oq bor edi. Abbos ixtiyoriy 3 tasini olmoqchi bo'ldi. Buni u necha xil usulda amalga oshirishi mumkin?

569. Zalda 2 ta bo'sh joy bor. 3 nafar kishidan 2 tasini shu joyga necha xil usulda o'tqazish mumkin?

570. Zilola 6 ta masaladan ixtiyoriy 4 tasini tanlamoqchi. Nazokat esa 6 ta boshqa masaladan 2 tasini tanlamoqchi. Zilola bu ishni necha xil usulda bajarishi mumkin? Nazokat-chi?

571. 7 ta olma va 3 ta nok bor. Ularni necha xil usul bilan har birida 5 tadan meva bo‘lgan va ulardan hech bo‘lmaganda 1 tasida nok bo‘lgan ikkita taqsimchaga qo‘yish mumkin?

572. Idishda 1, 2, 3, ..., 10 sonlari yozilgan sharlar bor. Idishdan uchta shar olamiz. Nechta holda ularda yozilgan sonlar yig‘indisi 9 ga teng bo‘ladi? Nechta holda 9 dan katta bo‘ladi?

573. 3 ta tovuq, 4 ta o‘rdak va 2 ta g‘oz bor. Bir nechta parrandani shunday tanlab olingki, ular ichida tovuq, o‘rdak va g‘oz bo‘lsin. Shunday tanlashlar soni nechta bo‘ladi?

574. 4 ta oq atirgul, 5 ta qizil va 3 ta sariq atirgul bor. Bir nechta gulni shunday tanlab olingki, ular ichida oq, qizil va sariq atirgul bo‘lsin. Shunday tanlashlar soni nechta?

575. 1, 2, 3, ..., 8 raqamlaridan ularni takrorlamay tuzilgan 8 xonali sonlar ichida 1 va 8 raqamlari yonma-yon turadiganlari nechta?

576. Gul sotuvchida 5 ta qizil va 10 ta oq chinnigul qolibdi. A’zamxon singlisi Mubinabonuga 2 ta qizil va 3 ta oq chinniguldan iborat guldasta sovg‘a qilmoqchi. Buni u necha xil usul bilan amalga oshirishi mumkin?

577. Tadbirkor 8 ta gazetadan 5 tasiga o‘z firmasi haqida e’lon bermoqchi. U 5 ta gazetani necha xil usulda tanlashi mumkin?

578. Aylanada yotuvchi 20 ta turli nuqta belgilandi. Uchlari belgilangan nuqtalarda yotuvchi: 1) vatarlar sonini; 2) uchburchaklar sonini; 3) qavariq to‘rtburchaklar sonini hisoblang.

579. Ikkita parallel chiziqning birida 8 ta, ikkinchisida 11 ta nuqta belgilandi. Uchlari belgilangan nuqtalarda bo‘lgan qavariq to‘rtburchaklar sonini toping.

580. Tepalikdagi buloqqa 6 ta yo‘l olib boradi. Sayyoh necha xil usulda buloqqa borishi va pastga tushishi mumkin? Agar

sayyoh buloqqa borgan yo‘lidan emas, boshqa yo‘ldan pastga tushsa, u holda tepalikka chiqish va undan tushish jami necha xil usulda bo‘lishi mumkin?

O‘zingizni tekshirib ko‘ring!

1. Futbol chempionatida 18 ta jamoa qatnashyapti. Agar har bir jamoa boshqa jamoa bilan o‘z maydonida va raqib maydonida o‘ynaydigan bo‘lsa, chempionatda jami qancha o‘yin o‘ynaladi?

2. 7- sinfda 12 ta fandan dars o‘tiladi. Dushanba kuni jadval bo‘yicha 5 soat dars bo‘lib, har bir soatda har xil dars o‘tiladi. Dushanba kungi jadvalni necha xil usulda tuzish mumkin?

3. 5 ta stulga 3 nafar o‘quvchini necha xil usulda o‘tqazish mumkin?

4. Matematikaga oid 5 ta turli kitobni javondagi 5 ta o‘ringa necha xil usulda qo‘ysa bo‘ladi?

VI bobga doir mashqlar

581. Agar: 1) raqamlar takrorlanmasa; 2) raqamlar takrorlanishi mumkin bo‘lsa 0, 1, 2, 3, 4, 5 raqamlaridan jami nechta 4 xonali son tuzsa bo‘ladi?

582. 0, 3, 4, 5, 6, 7 raqamlaridan jami nechta 4 xonali toq son tuzish mumkin?

583. Stolda ona tili, algebra, geometriya, ingliz tili darsliklari yotibdi. Malohat ularni kitob javoniga qo‘ymoqchi. Bu darsliklar javonda jami necha xil usulda turishi mumkin?

584. Odatda, uchburchakning uchlari lotin alifbosining katta harflari bilan belgilanadi. Lotin alifbosida 26 ta harf bor. Uchburchakning uchlarini necha xil usulda belgilash mumkin?

585. 8 ta stulga 3 nafar o‘quvchini necha xil usulda o‘tqazsa bo‘ladi?

586. Mijozning uy telefoni 7 raqamli bo‘lib, 218 dan boshlanadi. Mijoz a’zo bo‘lgan bu telefon stansiyasi nechta mijozga xizmat ko‘rsata oladi?

587. Necha xil usulda 5 nafar qilichbozdan 2 tasini musobaqada qatnashish uchun tanlab olish mumkin?

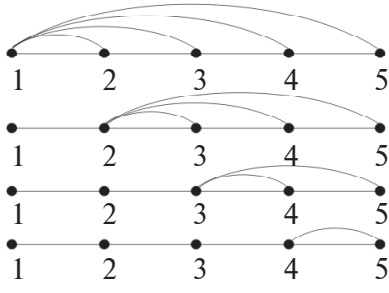
Alining yechimi: 5 nafar qilichbozdan bittasini tanlash imkoniyati 5 ta. 4 nafar qilichboz qoladi. Ulardan bittasini 4 usulda tanlasa bo‘ladi. Demak, $5 \cdot 4 = 20$.

Javob: 5·4=20 xil usul bor.

Nozimaning yechimi: 5 nafar qilichbozni „nomerlab“ chiqamiz va ulardan 2 kishilik guruhlar tuzamiz: 12; 13; 14; 15; 23; 24; 25; 34; 35; 45.

Javob: 10 xil usulda tanlash mumkin.

Mubinabonuning yechimi:



4 ta juftlik: 12; 13; 14; 15;

3 ta juftlik: 23; 24; 25;

2 ta juftlik: 34; 35;

1 ta juftlik: 45.

Jami 4+3+2=10. *Javob:* 10 xil usulda.

Kimning yechimi to‘g‘ri? Kimning yechimi sizga yoqdi? Nimasi bilan yoqdi?

588. Sizning tengdoshingiz bo‘lgan bir bola: „Hozircha men bir havaskor bolaman, katta bo‘lsam katta shoir bo‘laman“, deb yaxshi niyat qilib she’r yozib yurarkan. She’rlarining biriga „Lola“ deb sarlavha qo‘yibdi. Bu she’rning 1-qatori „Navbahorda qirda ochildi lola“ ekan. Qolgan qatorlar 1-qatordagi so‘zlarning o‘rnini almashtirish natijasida hosil bo‘lgan. Bu „she’r“ da eng ko‘pi bilan nechta qator bor?

589. Do‘kondagi 10 xil mevadan 3 xilini sotib olmoqchisiz. Buni necha xil usulda bajara olasiz?

590. Telefon stansiyasi telefonining nomeri 6 xonali son bo‘lgan 450 000 mijozga xizmat ko‘rsatadi.
1) Bu stansiya yana nechta mijozga xizmat ko‘rsata oladi?
2) Tarmoqqa yana 62 000 mijoz ulanishi mumkinmi?

591. To‘g‘ri chiziqda: 1) 4 ta; 2) 6 ta; 3) 10 ta; 4) n ta nuqta belgilandi. Har bir holda nechta kesma hosil bo‘ladi?

592. Aylana chizing va unda 4 ta nuqtani belgilang. Nechta yoy hosil bo‘ldi? Yoylarni turli rangdagi qalamlar bilan bo‘yang. Bunday qalamlardan nechta kerak bo‘ladi?

593. „Rayhon“ kafesining taomnomasida 3 xil somsa, 4 xil 1-taom, 5 xil 2-taom bor ekan. 3 xil turdagi taomga buyurtmani nechta usulda berish mumkin?

594. 2 ta olma, 2 ta nok, 2 ta shaftoli bor. 3 nafar o‘rtoq mevalarni har biri 2 ta turli meva oladigan qilib bo‘lib olishmoqchi. Buni jami nechta usulda bajarsa bo‘ladi?

595. „Navro‘z“ bayrami kunlarida kiyish uchun Oydin 4 xil adras ko‘ylakning biror xilini, 5 xil atlas ko‘ylakning ikki xilini tanlamoqchi. Oydin ko‘ylaklarni jami necha xil usulda tanlashi mumkin?

596. Hamma raqamlari: 1) juft bo‘lgan; 2) toq bo‘lgan nechta 5 xonali son bor?

VI bobga doir sinov mashqlari — testlar

1. 5 ga bo‘linadigan 6 xonali sonlar nechta?

A) $18 \cdot 10^4$; B) $9 \cdot 10^4$; C) $5 \cdot 6!$; D) $6 \cdot 5^4$.

2. Raqamlar takrorlanishi mumkin bo‘lsa, 1, 2, 3, 4, 5, 6, 7, 8 raqamlardan nechta 5 xonali son tuzish mumkin?

A) 8^5; B) 5^8; C) $8^2 \cdot 5^3$; D) $5^4 \cdot 8$.

3. Ikkita parallel to‘g‘ri chiziq berilgan bo‘lib, ularning birida 4 ta, ikkinchisida 3 ta nuqta belgilangan. Uchlari shu nuqtalarda bo‘lgan nechta uchburchak bor?

A) 30; B) 33; C) 40; D) 32;

4. 3 nafar o‘quvchini 6 ta stulga necha xil usulda o‘tqazish mumkin?

A) 120; B) 130; C) 100; D) 480.

5. Futbol jamoasidagi 11 kishi orasidan jamoa sardori va uning yordamchisini necha xil usulda tanlab olish mumkin?

A) 110; B) 55; C) 22; D) 121.

6. Bog‘iston qishlog‘idan Toshkentga 2 ta yo‘l bilan, Toshkentdan Urganchga 4 ta yo‘l bilan borish mumkin. Bog‘istondan Urganchgacha borish yo‘llari soni nechta?

A) 8; B) 10; C) 6; D) 12.

7. 12 ta oq atirgul va 13 ta qizil atirgulдан ikkita oq atirgul va uchta qizil atirgulдан iborat guldasta tuzish kerak. Buni necha xil usulda bajarish mumkin?

A) 18 876; B) 156; C) $12^2 \cdot 13^3$; D) 25.

8. Matematika to‘garagida faol qatnashuvchi 10 ta o‘quvchidan 4 tasini Xalqaro matematika olimpiadasiga yuborish uchun ularni necha xil usulda tanlasa bo‘ladi?

A) 210; B) 200; C) 40; D) 10^4.

9. Bir o‘quvchida qiziqarli matematikaga oid 7 ta kitob, ikkinchi o‘quvchida esa 9 ta badiiy kitob bor. Ular necha xil usul bilan birining bitta kitobini ikkinchisining bitta kitobiga ayirboshlashi mumkin?

A) 63; B) 49; C) 81; D) 126.

10. Otabekning tug‘ilgan kuniga uni tabriklash uchun 9 ta do‘sti keldi. Otabek ularning hammasi bilan, do‘stlari ham o‘zaro qo‘l berib ko‘rishishdi. Jami qo‘l berib ko‘rishishlar soni nechta?

A) 45; B) 90; C) 10; D) 50.

7- SINF ALGEBRA KURSINI TAKRORLASH UCHUN MASHQLAR

597. Sonli ifodaning qiymatini toping:

1) $2\frac{7}{8}+5\frac{5}{6}+7\frac{1}{8}+\frac{5}{6}$; 2) $13\frac{5}{6}\cdot\frac{1}{7}+\frac{1}{6}\cdot\frac{1}{7}$.

598. Tenglik to‘g‘rimi:

1) $\dfrac{2-\frac{3}{5}+0,7}{1\frac{4}{5}-1+0,4}=\dfrac{7}{4}$; 2) $\dfrac{\left(\frac{4}{7}-7-0,2\right)\cdot 3,5}{2,26}=-10$;

3) $\left(\dfrac{4,752}{3,2}+\dfrac{0,608}{3,8}\right):\left(7,5-\dfrac{3,55}{1,42}\right)=0,0617$?

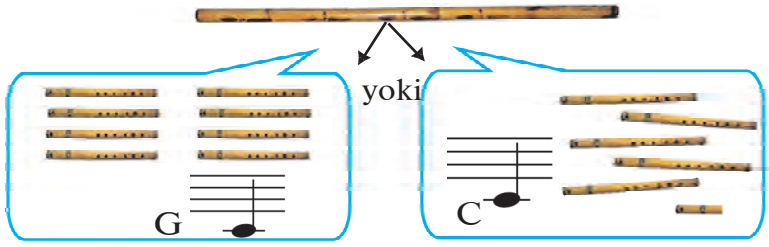
599. Ikki sondan biri a ga teng, ikkinchisi undan 7 ta ortiq. Shu sonlar ko‘paytmasining ikkilanganini toping. Shu ko‘paytmaning qiymatini $a=\frac{1}{2}$ bo‘lganda hisoblang.

600. Ikki sonning yig‘indisi 30 ga teng. Sonlardan biri a. Shu sonlarning ikkilangan ko‘paytmasini yozing. Shu ko‘paytmaning qiymatini $a=-2$ bo‘lganda hisoblang.

601. a ta yuzlik, b ta o‘nlik va c ta birlikdan tuzilgan natural sonda nechta birlik borligini ko‘rsatuvchi formula tuzing. Xuddi shu raqamlar yordamida, lekin teskari tartibda yozilgan sonda nechta birlik bor?

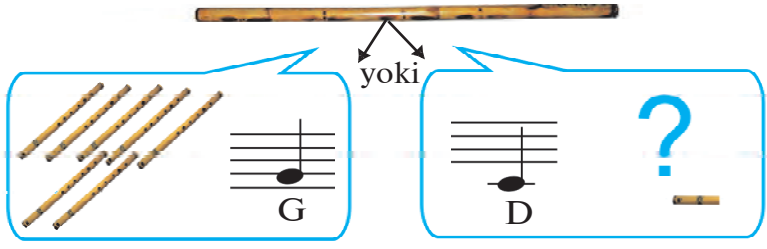
602. a kilogramm va c gramm necha grammni tashkil qiladi? Grammlar sonini x harfi bilan belgilab, javobni formula bilan yozing.

603. Qamishdan har birining uzunligi 6 sm bo‘lgan 8 ta hushtak yasashdi. Xuddi shunday uzunlikdagi qamishdan ikkinchi gal 5 ta hushtak yasashdi. 3 sm qamish bo‘lagi ortib qoldi (28- betdagi a- rasm). Ikkinchi gal yasalgan hushtakning uzunligi necha santimetr?



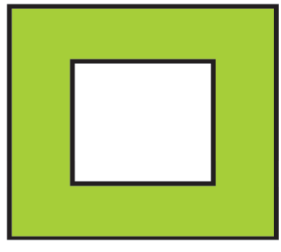
28- rasm.

604. Qamishdan har birining uzunligi 6 sm bo‘lgan 7 ta hushtak yasashdi. Xuddi shunday uzunlikdagi qamishdan ikkinchi gal bir nechta hushtak yasashdi, bunda 2 sm qamish bo‘lagi ortib qoldi (29- rasm). Ikkinchi gal nechta hushtak yasalgan bo‘lishi mumkin? (Hushtak uzunligi natural son va ≥ 3 sm.)



29- rasm.

605. 30- rasmdagi ichki kvadratning tomoni tashqi kvadrat tomonidan 20 sm qisqa. Bo‘yalgan sohaning yuzi 800 sm^2 bo‘lsa, kvadratlarning tomonlarini toping.



30- rasm.

606. Ifodani soddalashtiring:

1) $2a^2+2ab+3b^2-a^2-2b^2$;

2) $7a^2+2b^2-\left(6a^2+b^2\right)$;

3) $\frac{2}{3}a^2-b^2+\frac{4}{3}a^2-\frac{5}{7}b^2$;

4) $\frac{1}{7}a^2b\cdot 23m-\frac{2}{7}a^2bm$.

607. Ifodaning son qiymatini toping:

1) $5a^2-2ab+6a-7ab-6a^2-6a$, bunda $a=5$, $b=-\frac{1}{9}$.

608. Ko‘phadni birhadga ko‘paytiring:

1) $\left(a^2-ab+b^2\right)\cdot 3ab^3$;

2) $\left(6a^2-4ab^2+1\right)\cdot\frac{1}{2}ab$.

609. Ko‘phadlarni ko‘paytiring:

1) $\left(a^2+3ab+b^2\right)(7a-5b)$;

2) $(a+3b-4c)(a-3b-4c)$;

3) $\left(\frac{1}{3}a^2b-\frac{2}{5}ab^2\right)(15a-30b)$;

4) $\left(\frac{1}{2}a^2+4a+1\right)(3a-1)$.

Tenglamani yeching **(610—614):**

610. 1) $4(2x-1)+3(1-2x)=7$;

2) $4(x+2)-2(3x-2)=14x-5(x+3)$.

611. 1) $\frac{x-2}{4}-\frac{1}{2}=\frac{x+7}{6}$;

2) $\frac{2(3x-1)}{5}=4-\frac{x+2}{2}$.

612. 1) $7-\frac{x}{2}=3+\frac{7x}{2}$;

2) $\frac{x+3}{2}=x-4$.

613. 1) $\frac{x}{2}+\frac{x}{3}+\frac{x}{6}=12$;

2) $\frac{2x-1}{5}-\frac{x+1}{5}=\frac{3(1-x)}{10}$.

614. 1) $\frac{6x+7}{7}+\frac{3+5x}{8}=3$;

2) $5-\frac{2x-5}{3}=\frac{4x+2}{3}$;

3) $1+x=\frac{5x-2}{2}$;

4) $\frac{1-x}{9}-1=7x$.

615. Uchta qutida 119 ta qalam bor. Birinchi qutida ikkinchidagiga qaraganda 4 ta ortiq va uchinchidagiga qaraganda 3 ta kam qalam bor. Har bir qutida nechtadan qalam bor?

616. Otasi 30 yoshda, o‘g‘li esa 4 yoshda. Necha yildan keyin otasi o‘g‘lidan uch marta katta bo‘ladi?

617. O‘g‘li 6 yoshda, otasi esa undan 6 marta katta. Necha yildan keyin o‘g‘li otasidan 4 marta yosh bo‘ladi?

618. Ikki velosipedchi bitta yo‘l ustidagi qishloqlardan bir-biriga qarab bir vaqtda yo‘lga chiqdi. Birinchisi 15 km/soat, ikkinchisi esa 12 km/soat tezlik bilan harakat qilmoqda. Agar qishloqlar orasidagi masofa 40,5 km bo‘lsa, ular qancha vaqtdan keyin uchrashadilar?

619. Ikki velosipedchi bir yo‘ldagi ikkita qishloqdan bir vaqtda bir xil yo‘nalishda yo‘lga chiqdi. Ikkinchi velosipedchi oldinda, birinchisi orqada bormoqda. Birinchi velosipedchining tezligi 15 km/soat, ikkinchisiniki esa 12 km/soat. Agar qishloqlar orasidagi masofa 4,5 km bo‘lsa, birinchi velosipedchi ikkinchisini qancha vaqtda quvib yetadi?

Soddalashtiring **(620—622):**

620. 1) $(a+1)(a-1)(a^2+1)$; 2) $\left(\frac{a}{2}-5\right)\left(5+\frac{a}{2}\right)+25$.

621. 1) $(a+3)^2+(a-3)^2$; 2) $(4a+b)^2-(4a-b)^2$.

622. 1) $(1-a)(1+a+a^2)+a^3$;

2) $\left(\frac{1}{2}-c^2\right)\left(\frac{1}{4}+\frac{1}{2}c^2+c^4\right)+c^6$.

Ko‘paytuvchilarga ajrating **(623—624):**

623. 1) $a^4+6a^3+9a^2$; 2) $25-(2-3a)^2$.

624. 1) $(a+1)^2-(4-3a)^2$; 3) $(2a+b)^2-9(a+b)^2$;

2) $(8b-1)^2-(2b+3)^2$; 4) $4(a-2b)^2-25(3a-b)^2$.

625. Kasrni qisqartiring:

1) $\frac{a^2-16}{a^2-8a+16}$; 2) $\frac{4x^2-9}{2x+3}$.

Amallarni bajaring **(626—629):**

626. 1) $\frac{b+3}{5}+\frac{7+b}{10}+\frac{b-3}{2}$; 2) $\frac{a^2+5a-4}{16-a^2}+\frac{2a}{8a+2a^2}$.

627. 1) $\frac{a}{a^2-1}-\frac{1}{1-a^2}$; 2) $\frac{4x^2}{2x-3y}+\frac{12xy}{3y-2x}+\frac{9y^2}{2x-3y}$.

628. 1) $\frac{a-b}{ab}-\frac{a-c}{ac}$; 2) $\frac{1}{14x^3}-\frac{1}{21x^2y}+\frac{1}{4xy^2}$.

629. 1) $\frac{x^2-y^2}{6xy}\cdot\frac{12x^2y}{x+y}$; 2) $\frac{a^2+4a}{a^2-16}:\frac{4a+16}{a^2-4a}$.

Amallarni bajaring **(630—632)**:

630. 1) $\left(\frac{a}{a+1}+1\right):\left(1-\frac{a}{a+1}\right)$; 2) $\frac{1-a^2}{1+b}\cdot\frac{1-b^2}{a+a^2}\cdot\left(1+\frac{a}{1-a}\right)$.

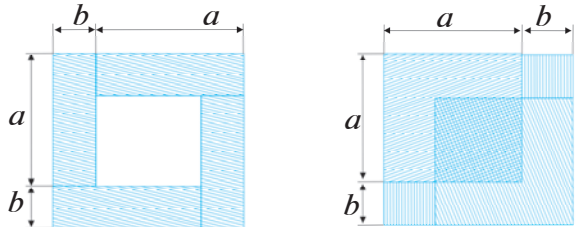
631. 1) $1+3a+\frac{9a^2}{1+3a}+\frac{1}{3a-1}+\frac{6a}{1-9a^2}$;

2) $\left(\frac{a+b}{a-b}+\frac{a-b}{a+b}\right):\left(\frac{a^2+b^2}{a^2-b^2}+\frac{a^2-b^2}{a^2+b^2}\right)$.

632. 1) $\left(\frac{9m^2-3n^2}{4m^2}-\frac{m-4n}{5m}\right):\left(\frac{2m+n}{3m}-\frac{5n^2-3m^2}{16m^2}\right)$;

2) $\left(\frac{a+4b}{2b}+\frac{6b}{4b-a}\right)\left(1-\frac{a^2-2ab+4b^2}{a^2-4b^2}\right)$.

633. 31- rasmlarning: 1) $(a+b)^2-(a-b)^2=4ab$;
2) $(a+b)^2+(a-b)^2=2(a^2+b^2)$ tengliklarga qanday aloqasi bor?



31- rasm.

634. Sayyoh Ko‘ksuv daryosi bo‘yida joylashgan bir oromgohdan velosipedda yo‘lga chiqib, boshqa bir oromgohga tayinlangan vaqtda yetib bormoqchi bo‘ldi. Dastlabki 1 soatda u 10,5 km yo‘l bosdi. Agar qolgan masofani ham shunday tezlik bilan o‘tsa, manzilga mo‘ljallagan vaqtdan 1 soat kechikishini hisoblab bildi. Sayyoh qolgan yo‘lni soatiga 15 km tezlik bilan o‘tdi va manzilga belgilangan vaqtdan yarim soat oldin yetib keldi. Oromgohlar orasidagi masofani toping.

635. Hozir soat 5. Qancha vaqtdan so‘ng soatning minut mili soat milini „quvib yetadi“?

636. Ikki xonali sonning o‘nliklar xonasidagi raqam birliklar xonasidagi raqamdan 4 marta katta. O‘quvchi 507 ni shu ikki xonali songa ko‘paytirmoqchi edi. Ammo u ikki xonali sonning raqamlari o‘rnini almashtirib yozib qo‘ydi. Natijada, u topgan ko‘paytma masalaning javobidan 27 378 ga kichik chiqdi. To‘g‘ri javob nechaga teng ekan?

637. Mis va ruxdan iborat qotishmaning og‘irligi 36 N ga teng. Qotishmani suvga botirilganda u o‘z og‘irligining $4\frac{1}{3}$ N ini yo‘qotdi. Mis suvga botirilganda o‘z og‘irligining $11\frac{1}{9}$ % ini, rux esa $14\frac{2}{7}$ % ini yo‘qotishi ma’lum. Qotishmadagi mis va rux og‘irligini aniqlang.

638. Tarkibi kumush va misdan iborat qotishmaning massasi 3,5 kg. Undagi kumush tarkibi mis tarkibining $16\frac{2}{3}$ % ini tashkil qiladi. Qotishmadagi kumush massasini toping.

639. 3 ta qopda 120 kg un bor. 1- qopdagi un 2- qopdagi unning $\frac{3}{5}$ qismiga, 3- qopdagi un esa 2- qopdagi unning 80 % iga teng. Har bir qopda necha kilogramm un bor?

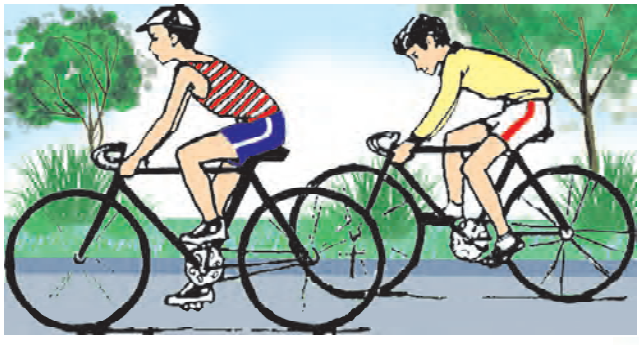
640. Ahmad *A* qishloqdan *B* qishloqqacha velosipedda 14 km/soat tezlik bilan, qaytishda esa 10 km/soat tezlik

bilan yurdi. Agar Ahmad qaytishga 1 soat ortiq vaqt sarflagan bo‘lsa, qishloqlar orasidagi masofani toping.

641. Vertolyot ikki qishloq orasidagi masofani shamol yo‘nalishida 1,5 soatda, shamol yo‘nalishiga qarshi esa 2 soatda uchib o‘tadi. Agar shamolning tezligi 10 km/soat bo‘lsa, shu qishloqlar orasidagi masofa qancha?

642. Firma reja bo‘yicha bir nechta mahsulotni 10 kun muddat ichida tayyorlashi kerak edi. Lekin u har kuni rejaga qo‘shimcha 2 tadan mahsulot tayyorlab, muddatga bir kun qolganda faqat topshiriqni bajaribgina qolmasdan, balki rejadan yana 3 ta ortiq mahsulot tayyorladi. Firma reja bo‘yicha 10 kunda nechta mahsulot tayyorlashi kerak edi?

643. 1) 7-sinfning ikki o‘quvchisi Ahmad va Karim velosiped poygasida qatnashishdi. Ahmad 15 km/soat tezlikda, Karim esa 18 km/soat tezlikda velosiped haydadi. Karim marraga Ahmaddan 20 minut avval keldi. Poyga masofasi necha kilometr ekan?



2) Sayyoh yo‘lning yarmini o‘tgach, dam oldi. So‘ngra yo‘lning 0,4 qismini yurdi. Hisoblab ko‘rsa, u 27 km yo‘l yurib qo‘yibdi. Mo‘ljallangan yo‘l jami necha kilometr ekan?

644. *(Al-Xorazmiyning masalalaridan.)*

1) Biri ikkinchisidan 2 ta ortiq sonlarning nisbati $\frac{1}{2}$ ga teng. Shu sonlarni toping.

2) Bir odam shunday vasiyat qildi: naqd 10 dirham (pul birligi) pulim bor. Bir kishiga qarz ham berganman. Qarzning miqdori o‘g‘lim oladigan merosga teng. Ikkala o‘g‘lim barobar meros olsin. Ukamga jami merosning $\frac{1}{5}$ qismini va yana 1 dirham beringlar. U kishining o‘g‘illari va ukasi necha dirhamdan olishgan?

Amallarni bajaring **(645—648):**

645. 1) $\left(\frac{c-d}{c^2+dc}-\frac{c}{d^2+cd}\right):\left(\frac{d^2}{c^3-cd^2}+\frac{1}{c+d}\right);$

2) $\left(\frac{2n}{k+2n}-\frac{4n^2}{k^2+4nk+4n^2}\right):\left(\frac{2n}{k^2-4n^2}+\frac{1}{2n-k}\right);$

3) $\left(\frac{b^2}{b+x}-\frac{b^3}{b^2+x^2+2bx}\right):\left(\frac{b}{b+x}-\frac{b^2}{b^2-x^2}\right);$

4) $\left(\frac{2q}{2q+m}-\frac{4q^2}{4q^2+4mq+m^2}\right):\left(\frac{2q}{4q^2-m^2}+\frac{1}{m-2q}\right).$

646. 1) $1+a-\frac{a-1}{a}+\frac{a^2-1}{2a}-\frac{3a}{2};$

2) $\frac{m+1}{m^2+m+1}-\frac{2}{1-m}+\frac{3m^2+2m+4}{1-m^3};$

3) $\frac{m+n}{3}-m+2n;$ 4) $m+n-\frac{2m-n}{5}-\frac{m+n}{2}.$

647. 1) $\frac{a^3+2a^2}{a^2-1}\cdot\frac{(a+1)^3(a-1)}{a^2(a+2)};$ 2) $\frac{(a^2+ab)^2}{a^2-b^2}:\frac{(a+b)^2}{(ab-b^2)^2}.$

648. 1) $1{,}5\cdot\left(2b-\frac{3b}{7}\right)-1\frac{5}{7}\cdot(3b-5)+\frac{9b^2-16}{4-3b};$

2) $\frac{x+3a}{x+a}-\frac{x}{x-a}+\frac{2a^2-ax+x^2}{a^2x^2}:\frac{x^2-a^2}{a^2x^2}.$

Tenglamani yeching **(649—650):**

649. 1) $\frac{4x-3}{2}-\frac{5-2x}{3}-\frac{3x-7}{6}=0;$ 2) $\frac{x+4}{5}-\frac{x+3}{3}=x-5-\frac{x-2}{2}.$

650. 1) $(2x-3)(x+5)-(3-x)(5-2x)=-30;$

2) $5(x-1)^2-2(x+3)^2=3(x+2)^2.$

651. Avtomobil shahardan qishloqqacha bo‘lgan masofani 80 km/soat tezlik bilan bosib o‘tdi. Orqaga qaytishda u masofaning 75 % ini avvalgi tezlik bilan, qolgan yo‘lni esa 60 km/soat tezlik bilan bosib o‘tdi va shuning uchun ham qaytishda yo‘lga shahardan qishloqqa borishdagiga qaraganda 10 minut ortiq vaqt sarf qildi. Shahardan qishloqqacha bo‘lgan masofani toping.

652. Qayiq daryo oqimiga qarshi 4,5 soat va oqim bo‘yicha 2,1 soat suzdi. Qayiq hammasi bo‘lib 52,2 km suzdi. Agar daryo oqimining tezligi 3 km/soat bo‘lsa, qayiqning turg‘un suvdagi tezligini toping.

653. Oralaridagi masofa 340 km bo‘lgan ikki bekatdan bir vaqtda bir-birlariga qarab ikki poyezd yo‘lga chiqdi. Ulardan birining tezligi ikkinchisinikidan 5 km/soat ortiq. Agar harakat boshlanganidan 2 soat o‘tgandan keyin poyezdlar orasidagi masofa 30 km ekanligi ma’lum bo‘lsa, ularning tezligini toping.

654. Ifodaning son qiymatini toping:

1) $(x-y)(x+y)(x^2+y^2)-8x^3+9y^2$, bunda $x=2,\ y=3;$

2) $-\frac{2}{3}(x-1)^2-2\frac{1}{3}(x-3)(x+3)$, bunda $x=3.$

655. Nechta 4 xonali sonda faqat bitta 0 raqami bor?

656. 0, 1, 2, 3, 5, 8 raqamlaridan ularni takrorlamasdan jami nechta 3 xonali son tuzsa bo‘ladi?

657. Hisoblang: 1) C_{10}^4; 2) P_7.

658. 6 nafar mehmonni 6 ta stulga necha xil usulda o‘tqazish mumkin?

1. 2) 7; 4) 5,86. **2.** 2) $\frac{9}{56}$; 4) 0,5. **4.** 2) Noto‘g‘ri; 4) Noto‘g‘ri. **5.** $40 \cdot 0{,}03 = 6 : 5$. **6.** 2) $3 \cdot (2 + 6) = 2 \cdot (2 \cdot 6)$. **8.** 2) $\frac{9}{56}$; 4) $4\frac{6}{7}$; **9.** 2) $-0{,}02$; 4) 3. **10.** 2) 0; 4) 5. **11.** 2) -2; 4) 0. **12.** $(7m)$t; 168 t. **13.** 1) $(60m)$ min.; 2) $\frac{p}{60}$ min; 3) $(60m + l + \frac{p}{60})$ min. **14.** $3(x - y)$; 2) 4,5; 4) 2,5. **15.** $(x + y)(x - y)$; 2) $-\frac{11}{64}$; 4) 0,104. **16.** 2) $-1\frac{2}{3}$. **17.** 2) 4. **18.** 1, 3, 15, 21. **19.** 2) $(m - 1)m$; 4) $(2p + 1)(2p + 3)(2p + 5)$. **21.** $(p - q)$ t; 1) 5t; 2) q p dan katta bo‘lmaydi; q p ga teng bo‘lishi mumkin. **22.** $400n + 500m$; 155000; 155000. **24.** 187200 m^3, $(37440m)$ m^3. **25.** $s = 3\frac{1}{6}c + 1\frac{2}{3}a + 2\frac{1}{2}b$, 53 km. **26.** 2) $a-b$; 4) $2mn$; 6) $(a+b)(a-b)$. **28.** 5000; 150000. **29.** $3a$; $8a$; $10a$; 500; 400; $\frac{sa}{100}$. **30.** 2) 30 kg. **31.** 2) $(5k)$ km. **32.** $(50a)$ kg. **33.** $(15a)$ ga. **34.** $(x \cdot 6 + y \cdot 3)$ so‘m. **35.** $(a \cdot 15 + b \cdot 20)$ kg. **36.** $(km + cn)$ kg. **37.** $S = a(a - b)$. **38.** $mn + k$; 810 o‘rin. **39.** 4 soat 35 min. **40.** b) $p = (m + n) \cdot 2$; $S = mn - xy$; e) $p = 2(a + m + n + x)$, $S = mn - ab - xy$. **41.** 2) $2(2a+4)m$; 3) $(a+8)(a-4)m^2$. **42.** $\frac{s}{t-1}$ km/soat. **44.** $\frac{a-1500}{20}$ m^2. **45.** $500(100 + p)$ so‘m. **47.** $t = \frac{s-3}{v}$, ulgurmaydi. **49.** 2) 40; 4) -41. **50.** 2) $3y-2x$; 4) $8{,}7 - 2\frac{1}{3}m + 1\frac{2}{3}n$. **51.** 2) $3 - 2{,}7b$; 4) $\frac{2}{3}y + \frac{1}{3}b - 3$; 6) $5p$. **52.** 2) $x + 5$; 4) $58c + 14d$. **53.** 2) 67,048; 4) $-11{,}221$. **54.** 2) 0,28; 4) $7\frac{37}{112}$. **55.** 2) $-4-9+11$; 4) $2a-3b-4c$. **57.** 2) $2 + b + (-c)$; 4) $3 + a + (-b) + (-c)$. **58.** 2) $a-2b+3c$; 4) $-a + 2b - 3c$. **59.** 2) $a - b + c - d$; 4) $a - b - c + d - k$. **60.** 2) $8x - 2y$; 4) $3a - 3$. **61.** 2) $a - 2b + (m + c)$; 4) $a + (-m + 3b^2 - 2a^3)$. **62.** 2) $2a + b - (-m - 3c)$; 4) $a-(m - 3b^2 + 2a^3)$. **63.** 2) $a-(b-1)$; 4) $(a-2b)+8$. **65.** 2) $c + (-a + b)$; 4) $n+(-d+l)$. **66.** 2) $4a - 4b$; 4) $5x - 3y$. **67.** 2) $x = 1$; 4) $x = 5$. **68.** 2) $-1{,}16$; 4) -3. **69.** 2) -1; 4) 9; 6) 9; 8) 3,9. **70.** 2) 147; 4) 144. **71.** 2) -132; 4) 7. **72.** 2) 1,08; 4) 6,12. **73.** 2) 12; 4) -1. **78.** 6 dirham. **80.** 2) 3. **85.** 2) $x = -27$; 4) $x = 1{,}009$. **86.** 2) $x = \frac{5}{7}$; 4) $x = \frac{2}{3}$. **87.** 2) $x = -1{,}3$; 4) $x = 0{,}05$. **88.** 2) $x = 64$; 4) $x = 1$. **89.** 2) $x = -\frac{4}{25}$; 4) $x = -\frac{1000}{3}$. **90.** 2) $x = \frac{3}{7}$; 4) $x = \frac{1}{3}$. **91.** 2) $x = 17$; 4) $y = -1$. **92.** 2) $x = 7\frac{1}{2}$; 4) $y = 24$. **93.**

2) $z=6$; 4) $x=0{,}6$. **94.** 2) $y=13$; 4) $x=1$. **95.** 2) $y=319$; 4) $x=5$. **96.** 2) $x=37$; 4) $x=1{,}1$. **99.** 2) $x=1$; 4) $x=1$. **100.** 2) $x=0{,}2$; 4) $x=4$. **102.** 2) 12 kishi. **103.** 2) 144, 432, 216. **104.** 2) 8, 8, 6. **105.** 2) 20, 40. **106.** 25, 27, 29. **107.** 4, 6, 8 va 10. **108.** 2) Bir soatda 12 dona mahsulot **109.** 89,6 m. **110.** 7 ta. **111.** 2) 2 kg. **112.** 2) 40 kg. **113.** 2) 150 ta mashina. **115.** 1) 0,2 qismi; 2) 0,25 qismi. **116.** 83,6 kg, 508, 8 kg, 1327 kg. **117.** 8 km/soat. **123.** 2) $\left(\frac{1}{3}\right)^5$; 4) $(-2{,}7)^4$. **124.** 2) m^5; 4) $(-3b)^4$. **125.** 2) $(a+b)^2$; 4) $\left(\frac{m}{n}\right)^5$. **126.** 2) $4^4\cdot 21$; 4) $6^2\cdot 7^2\cdot 3^3$. **127.** 2) $(0{,}5)^3\cdot 2^2\cdot 4^2$; 4) $\left(\frac{2}{3}\right)^3\cdot(2,3)^2$. **128.** 2) $x^4\cdot 3^2$; 4) $\left(\frac{a}{b}\right)^2(8a-b)^3$. **129.** 2) a^2+b^4; 4) $2x^3$. **130.** 2) na^3; 4) 5^k+a^{17}. **132.** 2) 9; 4) 125. **133.** 2) -1; 4) 0. **134.** 2) $\frac{9}{25}$; 4) $12\frac{19}{27}$. **135.** 2) 2,89; 4) $\frac{1}{625}$. **136.** 2) -125; 4) $-5\frac{1}{16}$. **137.** 2) 270; 4) 4. **138.** 2) 40; 4) -6. **139.** 2) 18; 4) 72. **140.** $-2\frac{1}{4}$, $2\frac{1}{4}$, $-3\frac{3}{8}$; -25, 25, 125. **146.** 2) 7^6; 4) 5^6. **147.** 2) a^7; 4) $(3b)^7$. **148.** 2) $(-3)^4$; 4) $(-1{,}2)^7$. **149.** 2) 3^{10}; 4) $(-6)^{12}$. **150.** 2) $\left(\frac{2}{3}\right)^8$; 4) b^{15}. **151.** 2) $\left(\frac{-5x}{6}\right)^{12}$; 4) $(n+m)^{20}$. **152.** 2) 3^{8+n}; 4) a^{n+13}. **154.** 2) 2^2; 4) 2^7. **155.** 2) 2^6; 4) 2^{10}. **156.** 2) 2^{14}; 4) 2^9. **157.** 2) 2^{23}; 4) 2^{4+n}. **158.** 2) 3^1; 4) 3^4. **159.** 2) 3^5; 4) 3^7. **160.** 2) 3^{18}; 4) 3^6. **161.** 2) 3^{n+1}; 4) 3^{3+n}. **162.** 2) 4^2; 4) 10^8. **163.** 2) $\frac{1}{17}$; 4) d^{12}. **164.** 2) $(2a)^2$; 4) $(m+n)^5$. **165.** 2) 2^2; 4) 2^2. **166.** 2) 2^3; 4) 2^9. **167.** 2) 3^3; 4) 3. **168.** 2) 3^2; 4) 3^4. **169.** 2) 6; 4) 25. **170.** 2) 44; 4) 9. **171.** 2) -6; 4) 12. **172.** 2) $x=64$; 4) $x=27$. **173.** 2) $x=16$; 4) $x=4$. **174.** 2) $x=243$; 4) $x=9$. **175.** 2) a^{56}; 4) a^{21}. **176.** 2) a^{15}; 4) a^{23}. **177.** 2) a^9; 4) a^{12}. **178.** 2) $n=7$; 4) $n=2$. **179.** 2) $\left(\frac{5}{6}\right)^2$; 4) $(0{,}02)^2$. **180.** 2) $(7^3)^2$; 4) $\left(\left(-\frac{2}{3}\right)^{12}\right)^2$. **181.** 2) $(b^3)^2$; 4) $(x^{10})^2$. **182.** 2) $7^5\cdot 6^5$; 4) $4^3\cdot\left(\frac{1}{7}\right)^3$. **183.** 2) $81x^4$; 4) $64b^2$. **184.** 2) 6^6y^6; 4) $27n^3m^3$. **185.** 2) $x^7y^7z^7$; 4) $2^9\cdot 4^9\cdot 9^9$. **186.** 2) a^6b^3; 4) $0{,}01c^6$. **187.** 2) $512a^{12}b^{21}$; 4) $16n^4m^{12}$. **189.** 2) $(3{,}4\cdot b)^4$; 4) $\left(-\frac{2}{3}a\right)^2$. **190.** 2) $(9\cdot r)^2$; 4) $(15\cdot a\cdot b)^3$. **191.** 2) $(a^2b^3)^2$; 4) $(9m)^2$. **192.** 2) $(xy^2z^4)^2$; 4) $(10c^4x^3)^2$. **193.** 2) $(0{,}7nm^5)^2$; 4) $\left(\frac{4}{25}a^5b^8\right)^2$. **194.** 2) $(b^3)^3$;

4) $(4^2)^3$. **195.** 2) $\left(\left(-\frac{2}{3}\right)^5\right)^3$; 4) $(-0,1)^3$. **196.** 2) $(a^2b)^3$; 4) $(x^4y^3z^2)^3$. **197.** 2) $(-10b^2)^3$; 4)$(-0,2xy^3)^3$. **198.** 2) 1; 4) -1. **199.** 2) 1; 4) $\frac{1}{32}$. **200.** 2) 144; 4) 14. **201.** 2) 1; 4) 4. **202.** 2) 14; 4) 16. **203.** 2) $\frac{25}{49}$; 4) $\frac{b^3}{8^3}$. **204.** 2) $\frac{169}{n^2}$; 4) $-\frac{64}{c^3}$. **205.** 2) $\frac{81b^4}{625c^4}$; 4) $\frac{5^6}{7^{12}}$;. **206.** 2) $\frac{49}{(2+c)^2}$; 4) $\frac{(a+b)^7}{(a-b)^7}$. **207.** 2) $(\frac{2}{5})^5$; 4) $\left(\frac{5}{a}\right)^7$. **208.** 2) $\left(\frac{a}{b}\right)^3$; 4) $\left(\frac{7}{10}\right)^2$. **209.** 2) $\left(\frac{4x}{3y}\right)^4$; 4)$\left(-\frac{1}{3}\right)^3$. **212.** 1) $\cong 3{,}3\cdot10^5$ marta; 2) $\cong$9 yil. **213.** 2) $\frac{3}{10}$. **214.** 2) 3^{5n+2}; 4) b^{4n}. **215.** 2) 7; 4) 5. **216.** 2) $81x^8y^6z^{14}$; 4) $-2{,}48832a^{15}b^{10}c^{20}$. **217.** 2) a^2; 4) a^4. **218.** 2) $10^{20} > 20^{10}$; 4) $3^{40} > 6^{20}$. **220.** 2) $\frac{1}{3}$; 4) 13,2. **221.** 2) $8{,}647\cdot10^6$. **222.** 2) $3bc$; 4) ab^2. **223.** 2) $3a^2b$. **224.** 2) $100n$ (sm). **226.** 2) 8; 4) 1; 6) 18. **227.** 2) z^{11}; 4) m^4; 6) $72\,p^3q^2$; **228.** 2) 2. **229.** $\frac{12}{25}$ kun. **230.** 2) $6ab$; 4) $-2a^3$. **231.** 2) $35m^2n$; 4) $-4b^5$. **232.** 2) $-2m^3n$; 4) $\frac{5}{14}b^3c^2$. **233.** 2) $28x^3y^3$; 4) $2a^2b^2c^2$. **234.** 2) $-21a^6b^6c^2$; 4) $-\frac{9}{8}a^4x^3y^4$. **235.** 2) $-7{,}5m^7r^7n^5$; 4) $-7{,}5a^5b^7c^7$. **236.** 2) $-15m^3n^2$; 4) $-26a^4b^4c^5$. **237.** 2) $30a^4b^3$; 4) $4a^3b^2c^3$. **238.** 2) $25b^2$; 4) $4a^6$. **239.** 2) $16a^2b^2$; 4) $-8x^3y^3z^3$. **240.** 2) $-a^{10}b^5c^5$; 4) $16x^8y^{12}$. **241.** 2) $\frac{1}{81}m^8n^8$. **242.** 2) $-2a^4$; 4) $a^2b^5c^2y^2$. **243.** 2) x^5y^5; 4) $-4a^{10}b^{11}$. **244.** 2) $(4x^2)^2$; 4) $(9x^3y)^2$. **245.** 2) 204,8; 4) 1,008. **246.** $7\frac{1}{5}$ qarich. **250.** 2) $6a^2b^3 - 24a^4b$; 4) $-bc^5 + 5x^2y^4$. **251.** 2) $-6xy^4z - -20m^3n^2k^3$; 4) $\frac{1}{3}a^2b^2 - 2a^2b^3$. **252.** 2) 2; 4) 0. **253.** 2) $-7{,}6$; 4) -252. **254.** 2) $\frac{1}{3}y$; 4) $\frac{13}{16}a^2b$. **255.** 2) $2a+b$; 4) $2a^2-3b^2$. **256.** 2) $-y$; 4) $3{,}8a^2$. **257.** 2) a^2; 4) $2xy - -2{,}2y^2$. **258.** 2) $-\frac{7}{8}ab^2 + \frac{3}{8}a^2b$; 4) $4x-2{,}46y$. **259.** 2) $x^3 - x^2y - xy^2$. 4) $ab^2 + 2ab$. **260.** 2) $8b^2 - 19bc - 15c^2$; 4) $2x^2y$. **261.** 2) $-\frac{1}{3}a^2bc - 4a^2c$. **262.** 2) $3x + 3y$; 4) $3x + 1$. **263.** 2) $5a^2 - b^2$; 4) $-\frac{1}{2}b^2 + 1\frac{1}{4}$. **264.** 2) $0{,}1c^2$; 4) $6a + 22b$. **265.** 2) $-2a^2 - - 6ab + 6b^2$; 4)$25z + 30az^2$. **266.** 2) $-2b$; 4) $9x^3$. **267.** 2) $3x^2$; 4) $8a^2 - b^2 - ab$. **268.** 2) $-0{,}07x^2 + 0{,}06y^2$; $0{,}27x^2 - 0{,}1y^2$; 4) $0{,}61a^3 + 1{,}12b^3$; $1{,}39a^3 - 0{,}88b^3$. **269.** 2) $3x^2 + 3x^2y^2 - x^3$. **270.** 2) $-5b^2 + 3b$. **271.** 2) q^3; 4) $-5ab+8b^2$. **273.** $k+2m-n$.

274. 2) $1-\frac{1}{2}x$; 4) $20m-30n$. **275.** 2) $-10xz+8yz$; 4) x^3-x^2+x. **276.** 2) $75a^2b^2+15a^2b$; 4) $3x^2y^3-6x^4y^2$. **277.** 2) $16ab^2-24a^2bc+8abc^2$; 4) $x^3yz+2xy^3z+3xyz^3$. **278.** 2) $a^3b^7+\frac{3}{4}a^4b^4$. **279.** 2) $-3a+7b$; 4) $-14p-9$. **280.** 2) $-a^2b+6b^2$; 4) $19x-12$. **281.** 2) $2x-3{,}5$; 4) $0{,}5y-1{,}7$. **282.** 2) 5; 4) 204. **283.** 2) z^2+3z-4; 4) $bc+4c+5b+20$. **284.** 2) $-a^2+8a+20$; 4) $p-q+pq-q^2$. **285.** 2) $10a^2+7a-12$; 4) $20p^2-17pq+3q^2$. **286.** 2) $0{,}09-m^2$; 4) $0{,}04a^2-0{,}25x^2$. **287.** 2) $30x^4+30y^4-61x^2y^2$; 4) x^3+5x^2+7x+3. **288.** 2) $27a^3-8b^3$; 4) $27a^3+8b^3$. **290.** 2) $0{,}3x^2+xz-0{,}3y^2+yz$; 4) $0{,}3a^4-0{,}9a^3+2a^2+3a-10$. **291.** 2) $a^3-ab^2+3a^2b-3b^3$; 4) $12x^3-29x^2+7x+6$. **295.** 2) y^4; 4) 1. **296.** 2) $-3a$; 4) $-5c$. **297.** 2) $\frac{2}{15}a$; 4) $-9c$. **298.** 2) $9m$; 4) $\frac{4}{5}b$. **299.** 2) 8; 4) 7. **300.** 2) 3; 4) -3. **301.** 2) $-\frac{5}{3}$; 4) $-1{,}3$. **302.** 2) $-\frac{5}{3}p$; 4) $0{,}4c$. **303.** 2) $7m^6$; 4) $\frac{7}{6}$. **304.** 2) $\frac{9}{4}ab^2$; 4) $3ab$. **305.** 2) $-\frac{1}{13}axy^2$; 2) $\frac{1}{2}a^3b$. **306.** 2) $81x^4y$; 4) $x^7y^{11}z^3$. **307.** 2) $2b-1$; 4) $2-x$. **308.** 2) $4a-3b$; 4) $-c+1$. **309.** 2) $-\frac{2}{3}cb-1$; 4) $-\frac{1}{4}ab+\frac{3}{4}a^2$. **310.** 2) $-2x-3y+4$; 4) $a+3a^2b-2$. **311.** 2) 1; 4) $-3a$. **312.** 2) 200 m; 2400 m^2. **313.** 2) a^3; 4) c^2+3^2. **314.** 2) n^2-m^2; 4) $(\frac{1}{2})^3-b^3$. **315.** $4c$ sm, c^2 м². **317.** $3x^2$ yoki $\frac{1}{3}x^2$. **318.** 10 km. **319.** 108000. **320.** Yo‘q. **321.** 2) $3{,}08\cdot10^{13}$. **322.** $5{,}1\cdot10^8$; 10^{12}. **323.** 10 kg. **324.** 2) xy; 4) $10mn^2k$. **325.** 2) $13\frac{3}{4}$. **326.** 2) $3x^2$; 4) $8a^2+b^2-ab$. **327.** 2) $0{,}5x^2+xz-0{,}5y^2+yz$; 4) $a^4-2a^3+3a^2+4a-10$. **328.** 2) $2a^3-2ab^2+3a^2b-3b^3$; 4) $6x^3-17x^2-4x+3$. **329.** 2) $5x^3+8x^2+9x-1$; 4) $1\frac{1}{4}a^5+2a^2x-1\frac{1}{2}x^2$. **332.** 2) 180,7; 4) 12,5. **333.** 2) $2x^2-2x$; 4) $a^3+ab-a^2b^2-b^3$. **334.** 240 km. **336.** 2) $3(a-x)$; 4) $6(a+2)$. **337.** 2) $2(4a-2b-1)$; 4) $3(3x-y+4z)$. **338.** 2) $c(d+b)$; 4) $x(3-y)$. **339.** 2) $3b(d-a)$; 4) $3p(2k-1)$. **340.** 2) $x(y-x+z)$; 4) $4b(b+2a-3a^2)$. **341.** 2) $a^3(a-3)$; 4) $x^2y^2(y-x)$. **342.** 2) $6x^2(x^2-4)$; 4) $3a^2(2a^3+1)$. **343.** 2) $4x^2y(5xy+1)$; 4) $3xyz(3z-4y)$. **344.** 2) $5a^3(4a-1+3a^2)$; 4) $2x^2y^2(y^2-x^2+3xy)$. **345.** 2) 18700; 4) $-1{,}62$. **346.** 2) $(a+5)(b-c)$; 4) $(y-3)(1+b)$. **347.** 2) $(m-3)(3n+5m)$; 4) $(c-d)(7a-2b)$. **348.** 2) $(x+y)(a^2-b^2)$; 4) $(a^2-2b^2)(x+y)$. **349.** 2) $(p-q)(c-a+d)$; 4) $(x^2+1)(m-n-l)$. **350.**

2) $(b-c)(a+c)$; 4) $(x-y)(2b+1)$. **351.** 2) $(a-2)(6-a)$; 4) $(m-2)(a^2-b)$. **352.** 2) $(x-y)(x-y-3)$; 4) $(3-b)(-a+1-b)$. **353.** 2) $x=1$; 4) $x=0{,}49$. **354.** Ulguradi. **355.** 2) $(m-n)(1+p)$; 4) $(x-y)(1+2a)$. **356.** 2) $(a-b)(a-b+1)$; 4) $(p-1)(4q+p-1)$. **357.** 2) $(p-1)(4q+1)$; 4) $(p-1)(4q-1)$. **358.** 2) $(b+c)(a+d)$; 4) $2(x-1)(3x-4y)$. **359.** 2) $(c+d)(a-3b)$; 4) $(a-3b)(x+5y)$. **360.** 2) $(b+c-a)(y-x^2)$; **361.** 2) 12500; 4) 28. **362.** 2) $-0{,}625$; 4) $-0{,}33$. **363.** 2) 906. **364.** 2) $t=-7$, $t=4$. **365.** 2) $x^2-2xy+y^2$; 4) x^2+2x+1; 6) $49+14m+m^2$. **366.** 2) x^2-6x+9; 4) $y^2-12y+36$; 6) $b^2+b+\frac{1}{4}$. **367.** 2) $9x^2+12xy+4y^2$; 4) $25z^2-10zt+t^2$. **368.** 2) a^4+2a^2+1; 4) $x^4+2x^2y^2+y^4$. **369.** 2) $a^2-\frac{2}{3}a+\frac{1}{9}$; 4) $\frac{x^2}{9}+\frac{xy}{6}+\frac{y^2}{16}$. **370.** 2) $0{,}16b^2-0{,}4bc+0{,}25c^2$; 4) $\frac{1}{16}a^6-\frac{2}{5}a^3+\frac{16}{25}$. **372.** 2) $9b^4+12ab^3+4a^2b^2$; 4) $16x^2y^2+4xy^3+0{,}25y^4$. **373.** 2) 1681; 4) 9604. **374.** 2) 1006009; 4) 1521. **375.** 2) 3249; 4) 1002001. **376.** 2) $4xy$; 4) $8a^2+2b^2$. **377.** 2) $7a^2-52a+112$; 4) $4x^2-16x-4$. **378.** 2) $x=2$; 4) $x=-0{,}5$. **379.** 2) $y=3$; 4) $y=\frac{2}{3}$. **380.** 2) -11; 4) -17. **382.** 2) $(5+x)^2$; 4) $(p-0{,}8)^2$. **386.** 2) p^2-q^2; 4) m^2-n^2. **387.** 2) a^2-9; 4) x^2-49. **388.** 2) c^2-9d^2; 4) $9m^2-4n^2$. **389.** 2) $\frac{25}{36}a^2-b^2$; 4) $\frac{4}{9}m^2-\frac{9}{16}n^2$. **390.** 2) a^4-b^6; 4) m^6-n^6. **393.** 2) $25a^2b^4-4a^4b^2$; 4) $a^2b^6-16x^2y^2$. **394.** 2) x^4-1; 4) $81a^4-16b^4$. **395.** 2) 4896; 4) 2491. **396.** 2) 1584; 4) 39999. **397.** 2) $2a^2+4a$; 4) $24ab-32b^2$. **399.** 2) $x=\frac{4}{3}$; 4) $y=\pm 2$; $y=3$. **400.** 64 sm^2 ga kamaydi. **401.** -10. **402.** 2) 980; 4) 5,87. **405.** 2) $(2a-3)(2a+3)$; 4) $(9a-4b)(9a+4b)$. **406.** 2) $(ab-4)(ab+4)$; 4) $(4x-5y)(4x+5y)$. **407.** 2) $(\frac{2}{3}a-\frac{1}{4}b)(\frac{2}{3}a+\frac{1}{4}b)$; 4) $(0{,}3x-0{,}4y)(0{,}3x+0{,}4y)$. **408.** 2) $(xy^2-4)(xy^2+4)$; 4) $(5a-3b^3)(5a+3b^3)$. **409.** 2) $(a^2-b^4)(a^2+b^4)$; 4) $(b^2-9)(b^2+9)$. **410.** 2) $(m-n-k)(m-n+k)$; 4) $3(x-y)(3x+y)$. **411.** 2) $(a+2b+c)(a-c)$; 4) $4(2a-b)(-a-2b)$. **412.** 2) $(1+c)^2$; 4) $(9-x)^2$. **413.** 2) $(10-3a)^2$; 4) $(a+5b)^2$. **414.** 2) $(p^2-q)^2$; 4) $(5a^3+3b)^2$. **415.** 2) $(b^2-9)^2$; 4) $(4-a^2b^2)^2$. **416.** 2) $-(3-b)^2$; 4) $3(a+2b)^2$. **417.** 2) 60 000; 4) 216. **418.** 2) $x=\frac{1}{2}$, $x=-\frac{1}{2}$; 4) $x=5$.

419. 2) 10000; 4) $\frac{2}{3}$. **420.** 2) $x^2+2xy+y^2$; 4) $x^2-2xy+y^2$. **421.** $(c+d)(c^2-$ $-cd+d^2)$; 4) $(a-3)(a^2+3a+9)$; 6) $(a+1)(a^2-a+1)$; 8)$(5-b)(25+5b+b^2)$. **422.** 2) $(4-5y)(16+20y+25y^2)$; 4) $(4y+\frac{1}{3})(16y^2-\frac{4}{3}y+\frac{1}{9})$. **423.** 2) $(1+3b)(1-$ $-3b+9b^2)$; 4) $(\frac{1}{2}a^2+5b)(\frac{1}{4}a^4-\frac{5}{2}a^2b+25b^2)$. **424.** 2) $(a+b)(a-b)\times$ $\times(a^4+a^2b^2+b^4)$; 4) $(2+y)(2-y)(16+4y^2+y^4)$. **425.** 2) y^3+8; 4) $64c^3-125d^3$. **426.** 2) $a^6b^6-125a^3$; 4) $\frac{1}{8}x^3-\frac{1}{27}y^3$. **427.** 2) $16a^2(4a+5b)$; 4) $(a-b)(a^2+ab+b^2+$ $+a-b)$. **428.** 2) 0,02. **429.** 2) $8x+7$. **430.** 2) $x=3$; 4) $x=0{,}2$. **441.** 2) $x=2$. **442.** 2 km/soat, 16 km/soat. **443.** 2) $(x-y)(4+3x-3y)$; 4) $(b-a)(b-a-1)$. **444.** 2) $y(x+y)^2$; 4) $(b-a)^2(a-1)$. **445.** 2) $24x^2(y-z)$; 4) $4(2x-y)(2x-3y-1)$. **446.** 2) $5(x+y)(2x+1)$; 4) $(3z^2+2y^2)(16x-5y)$. **447.** 2) $(2nk+5m)(3mk-7n^2)$; 4) $(5c-3x)(8b-3c)$. **448.** 2) $16x+2$; 4) $-19y+6$. **450.** 2) $\frac{5}{8}$; 4) $\frac{11}{8}$. **454.** $\frac{a^2-b^2}{(a-b)^2}$. **456.** 2) 5; 4) 1,9; 6) 4. **457.** 2) $V=\frac{m}{p}$; 4) $a=\frac{p}{2}-b$. **458.** $x=\frac{np}{1000a}$, $x=3$. **459.** $t=\frac{a}{cn}$, $t=15$. **461.** 2) $\frac{4}{5}$, 4) -2. **462.** 2) $\frac{2}{3}$; 4) $\frac{b}{2c}$. **463.** 2) $\frac{1}{b^4}$; 4) b^2. **464.** 2) $\frac{2}{7}$; 4) $\frac{b}{3a}$; 6) $\frac{a^2b}{5c}$. **465.** 2) $\frac{7a}{5}$; 4) $\frac{1}{3(a-b)}$; 6) $-\frac{1}{3}$. **466.** 2) $\frac{1}{(m+n)^3}$; 4) $3y-2x$; 6) $\frac{2}{a(a-b)}$. **467.** 2) $\frac{2a}{m-n}$; 4) $\frac{4a-1}{2a+3}$; 6) $\frac{1+b}{1-b}$. **468.** 2) $\frac{q^2}{p-q}$; 4) $\frac{m}{n}$; 6) $-\frac{x}{y}$. **469.** 2) $\frac{3a+2b}{2a+3b}$; 4) $-\frac{1}{ab}$. **470.** 2) $\frac{1}{a+b}$; 4) $5+x$; 6) $-\frac{c+2}{2a}$. **471.** 2) $10-7b$; 4) $\frac{y}{5+y}$; 6) $\frac{5ab}{a^2-b^2}$. **472.** 2) $\frac{1}{b+7}$; 4) $\frac{1}{1-2p}$. **473.** 2) $\frac{4a+1}{4a-1}$; 4) $\frac{10(m+n)}{3(m-n)}$. **474.** 2) $n-m$; 4) $\frac{1}{5-2x}$. **475.** 2) $\frac{3y-4x}{3y+4x}$; 4) $\frac{6-c}{6+c}$; 6) $\frac{3c-2b}{a}$. **476.** 2) $a+1$; 4) $\frac{1}{2}$. **477.** 2) $\frac{b}{ab}$ va $\frac{2a}{ab}$; 4) $\frac{2a}{2b}$ va $\frac{a}{2b}$; 6) $\frac{32}{60}$ va $\frac{25}{60}$. **478.** 2) $\frac{9x^2}{12xy}$, $\frac{72}{12xy}$ va $\frac{16y^2}{12xy}$; 4) $\frac{2ax^2}{4x^3}$ va $\frac{b}{4x^3}$. **479.** 2) $\frac{6b^2}{2b}$ va $\frac{a^2}{2b}$; 4) $\frac{2b^2}{6ab}$, $\frac{9ac}{6ab}$, $\frac{6a^2b^2}{6ab}$. **480.** 2) $\frac{3a^2}{18a^2b^2}$, $\frac{2(a^2+b^2)}{18a^2b^2}$ va $\frac{a(3-a^2)}{18a^2b^2}$; 4) $\frac{21y^3}{60x^4y^4}$, $\frac{310x^3y}{60x^4y^4}$ va $\frac{80x^2}{60x^4y^4}$. **481.** 2) $\frac{6a}{(a-1)a}$ va $\frac{2(a-1)}{(a-1)a}$;

4) $\frac{8a^2}{12(a+1)}$ va $\frac{15a^2}{12(a+1)}$. **482.** 2) $\frac{7a(3x+y)}{9x^2-y^2}$ va $\frac{6b(3-y)}{9x^2-y^2}$; 4) $\frac{6x}{8x+8y}$ va $\frac{x}{8x+8y}$. **483.** 2) $\frac{7a}{x^2-9}$ va $\frac{a(x-3)}{x^2-9}$; 4) $\frac{6x(x+y)}{x^2-y^2}$, $\frac{7xy(x-y)}{x^2-y^2}$ va $\frac{3}{x^2-y^2}$. **484.** 2) $\frac{28c(b+c)}{70(b^2-c^2)}$, $\frac{6a^2}{70(b^2-c^2)}$ va $\frac{35b(b-c)}{70(b^2-c^2)}$; 4) $\frac{15x(x+1)}{12x(x^2-1)}$; $\frac{-48x^2}{12x(x^2-1)}$ va $\frac{4(x-1)}{12x(x^2-1)}$. **485.** 2) $\frac{5a}{b^3}$; 4) $\frac{x-y}{n+a}$. **486.** 2) $\frac{2a}{c^2}$; 4) $\frac{7}{a^2}$; 6) $\frac{8}{ab}$. **487.** 2) $\frac{11}{28}$; 4) $\frac{3}{5b}$; 6) $\frac{3ad-b}{12d}$. **488.** 2) $\frac{15+ab}{5a}$; 4) $\frac{2+7b}{b}$. **489.** 2) $\frac{2c+4c^2-3}{c^2}$; 4) $\frac{mn-kn^2+m^2}{n^2}$. **490.** 2) $\frac{k-n}{mnk}$; 4) $\frac{bd+ba}{acd}$; 6) $\frac{2n^2-3m}{mn^3}$. **491.** 2) $\frac{4a^4-21cb^3}{18a^3b^4}$; 4) $\frac{20y-21x+22}{28x^2y^2}$; 6) $\frac{b(cd^2+d+c)}{(cd)^2}$. **492.** 2) $\frac{3x}{2(1-x)}$; 4) $\frac{8y-25x}{10(y-3)}$. **493.** 2) $\frac{11}{10(b+1)}$; 4) $\frac{5x}{8(x+y)}$. **494.** 2) $\frac{5b^2-2a^2}{ab(x+y)}$; 4) $\frac{a+b-y}{ab}$. **495.** 2) $\frac{2(2a+3)}{a(1-a)}$; 4) $\frac{67b-3a}{40(a^2-b^2)}$. **496.** 2) $\frac{x-1}{x^2-9}$; 4) $\frac{2x^2+3x+2}{x^2-16}$. **497.** 2) $\frac{6n-47}{n^2-49}$; 4) $\frac{24y^2+y+1}{1-9y^2}$. **498.** 2) $\frac{13a+4}{(3a+1)^2}$. **499.** 2) $\frac{2-11x}{(3x+1)^2}$; 4) $\frac{4-7n+7m}{(n-m)^2}$; 6) $\frac{2x^2+18}{(x^2-9)^2}$. **500.** 2) $\frac{b^2-3b}{b-2}$; 4) $\frac{1}{a+1}$. **501.** 2) $-\frac{1}{x+y}$; 4) $\frac{2(24-a)}{4a^2-9}$. **502.** 2) $\frac{b-3b^2-14}{6(b^2-1)}$; 4) $\frac{28n^2-4m^2+9mn}{m(4n^2-m^2)}$; 6) $\frac{4a^2-4a-b}{a^2+2a}$. **503.** 2) $\frac{2a}{a^3+8}$; 4) $-\frac{6m}{m^3-27}$. **504.** 2) $-\frac{2}{19}$. **505.** 2) $\frac{4}{13}$; 4) $\frac{15}{2}$. **506.** 2) $\frac{k^2}{mn}$; 4) $\frac{3mk}{4nd}$; 6) $\frac{2a^2b^2}{c^3}$. **509.** 2) 2; 4) $\frac{a}{bc}$; 6) $\frac{ac}{b}$. **510.** 2) $\frac{k^2}{mn}$; 4) $\frac{3md}{2nk}$; 6) $\frac{15a^2c^2}{d}$. **511.** 2) $\frac{18a^2}{7}$; 4) $\frac{1}{a}$; 6) $\frac{a^3b^3}{d^2}$. **512.** 2) $\frac{2y}{5c^3}$; 4) $\frac{2d^2a^2}{3c}$; 6) $\frac{22p^3n}{m^4}$. **513.** 2) $10a^2b$; 4) $\frac{1}{4a^2b}$. **514.** 2) $\frac{2b}{a}$; 4) $3b$; 6) $\frac{(a+b)a}{3b}$. **515.** 2) $\frac{b}{3(1+a)}$; 4) $\frac{1}{3m^2(m+n)}$; 6) $\frac{5}{3(a-b)}$. **516.** 2) $\frac{-3x^2(x+y)}{2(x^2+y^2)}$; 4) $\frac{-18(n-m)^2(n+m)}{n(n+p)^2}$;

6) $\frac{1}{a^2-b^2}$. **517.** 2) $b-3$; 4) $(a-1)(2a-1)$. **518.** 2) $\frac{2(a+1)}{3}$; 4) 1; 6) $\frac{b^2}{b^2+1}$. **519.** 2) $\frac{a^2(b^2-1)}{b^2}$; 4) $\frac{2(m+n)}{n}$. **520.** 2) $\frac{4ab}{a^2-b^2}$; 4) $\frac{1}{6(c+d)}$. **521.** 2) $\frac{9z}{z+2}$; 4) $\frac{m+5}{m-2}$. **522.** 2) $\frac{b}{a+b}$; 4) $\frac{1}{c}$. **523.** 2) $\frac{4}{a-b}$; 4) $\frac{1}{c(a+b)}$. **526.** $\frac{v-v_1}{v+v_1}\cdot s$ km. **527.** 6 donadan. **528.** 2) $\frac{3(x^2-2x+4)}{x^3+8}$, $\frac{x+1}{x^3+8}$ va $\frac{(x+2)^2}{x^3+8}$. **529.** 2) $\frac{55b-61}{24}$; 4) $\frac{5-27b}{36}$. **530.** 2) $\frac{7q-p}{3p-q}$; 4) $\frac{8a+8b-70}{2b-5}$. **531.** 2) $\frac{a^2-b^2}{7}$; 4) $\frac{m+n}{2(p^2-pc+c^2)}$. **532.** 2) $\frac{x(x+2)(x-3)}{(x-2)(x+3)(x^2+2)}$; 4) 1. **533.** 2) $-2(a-1)^2$; 4) $\frac{a^2+4}{4a}$. **534.** 120. **536.** d) $n(n-1):2$. **538.** 45. **539.** 2) 900. **541.** $16\cdot15\cdot14=3360$. **542.** 30. **543.** 1) 125; 2) 625. **545.** 24. **546.** 10. **547.** $12\cdot8\cdot7=672$. **548.** 1) $64\cdot49=3136$; 2) 8! **550.** 1) $4\cdot60$; 2) $24\cdot58$; 3) $36\cdot55$; jami 3612 usul. **551.** 6. **552.** 12. **554.** 20. **555.** 14 ta. **561.** 24 ta 4 xonali son tuzish mumkin. **562.** 24. **565.** 10. **566.** 45. **568.** 56. **569.** 6. **570.** $C_6^4=C_6^2=15$. **572.** $C_{10}^3-4=116$ holda yig‘indi 9 dan katta bo‘ladi. **573** $\left(C_3^1+C_3^2+C_3^3\right)\cdot\left(C_4^1+C_4^2+C_4^3+C_4^4\right)\cdot\left(C_2^1+C_2^2\right)=315$ ta. **576.** $C_5^2\cdot C_{10}^3=1200$. **578.** 1) $C_{20}^2=190$; 2) $C_{20}^3=1140$; 3) $C_{20}^4=4845$. **579.** $8\cdot C_{11}^2+11\cdot C_8^2=748$. **580.** 36; 30. **581.** 1) $5\cdot5\cdot4\cdot3=300$; 2) $5\cdot6\cdot6\cdot6=1080$. **582.** $5\cdot6\cdot6\cdot3=540$. **583.** $4\cdot3\cdot2\cdot1=24$. **584.** $26\cdot25\cdot24=15\,600$. **585.** $8\cdot7\cdot5=280$. **586.** 10 000. **588.** 24 ta. **589.** $10\cdot9\cdot8=720$. **590.** 2) mumkin emas. **591.** 1) 6; 2) 15; 3) 45; 4) $n\cdot(n-1):2$. **593.** $3\cdot4\cdot5=60$. **594.** 4. **595.** 40. **596.** 1) 2500; 2) 3125. **597.** 2) 2. **598.** 2) Noto‘g‘ri. **599.** $7\frac{1}{2}$. **600.** $2a(30-a)$; -128. **601.** $a\cdot100+b\cdot10+c$; $c\cdot100+b\cdot10+a$; a ta. **602.** $x=1000a+c$. **606.** 4) $3a^2bm$. **609.** 4) $1{,}5a^3+11{,}5a^2-a-1$. **610.** 2) $x=2\frac{5}{11}$. **614.** 4) $x=-\frac{1}{8}$. **615.** 40, 36, 43. **616.** 9 yildan so‘ng. **617.** 4 yildan so‘ng. **618.** 1,5 soatda. **619.** 1,5 soat-da. **620.** 2) $\frac{a^2}{4}$. **621.** 2) $16ab$. **623.** 2) $3(1+a)(7-3a)$. **624.** 2) $4(3b-2)(5b+1)$;

4) $(17a-9b)(b-13a)$. **634.** 63 km. **635.** $27\frac{3}{11}$ minutdan so‘ng. **636.** 41574. **637.** Mis — 25,5N; rux — 10,5 N. **638.** $\frac{1}{2}$ kg. **640.** 35 km. **641.** 120 km. **642.** 150 ta. **644.** 2) o‘g‘illari $5\frac{5}{6}$ dirhamdan, ukasi $4\frac{1}{6}$ dirham. **645.** 2) $\frac{2n(2n-k)}{2n+k}$; 4) $\frac{2q(m-2q)}{m+2q}$. **646.** 4) $\frac{m+7n}{10}$. **648.** 2) 1. **649.** 2) $x=6$. **650.** 2) $x=-\frac{25}{34}$; 4) $x=-6{,}5$. **651.** 160 km. **652.** 9 km/soat. **653.** 80 km/soat; 75 km/soat. **654.** 2) $-2\frac{2}{3}$.

„O‘zingizni tekshirib ko‘ring!“ topshiriqlariga javoblar

I bob. **1.** 1) 120,3; 2) $-3\frac{1}{6}$; **2.** $3x+4y$; $\frac{1}{3}$. **3.** $10a+15b$.

II bob. **1.** Ha, $x=-4$; **2.** 1) $x=\frac{1}{3}$; 2) $x=3$. **3.** 30 %.

III bob. **1.** 5^5; 3^2; 2^{12}; 6^5. **2.** $3b+d$. **3.** $-1{,}25\,a^4b^3c^2$; $0{,}7m-2n-1$. **4.** $3m^2-4$; $-3{,}8125$.

IV bob. **1.** $2a^2+12a$. **2.** 1) $y(x-2)$; 2) $(4a-9)(4a+9)$; 3) $3x^2\cdot(1-2x)$; 4) $(x-5)^2$; 5) $(x-1)(3+y)$; 6) $2(a-b)^2$. **3.** $(a-3b)(a+3)$; 8.

V bob. **1.** $b\neq 0$, $a\neq 1$, $b\neq -2$. **2.** 1) $\frac{1}{a}$; 2) $\frac{4ab}{a^2-b^2}$; 3) 4; 4) $\frac{a-b}{b}$. **3.** $\frac{1}{x-3}$; -3.

VI bob. **1.** $18\cdot 17=306$. **2.** $12\cdot 11\cdot 10\cdot 9\cdot 8=87480$. **3.** $5\cdot 4\cdot 3=60$. **4.** $1\cdot 2\cdot 3\cdot 4\cdot 5=120$.

Qiziqarli masalalarga javoblar

1. $99+9:9$. **2.**44 ta uchburchak, 10 kvadrat, 8 ta to‘g‘ri to‘rtburchak. **3.** 5 yoshda. **4.** 18 minut. **5.** 1) 6; 2) 3; 3) 4; 4) 9. **6.** 24 000 km. **7.** 6 ta **8.** 1) 7; 2) 4 o‘g‘il, 3 qiz. **9.** 10 metr. **10.** Mumkin emas.

MUNDARIJA

IV bob. KO'PHADNI KO'PAYTUVCHILARGA AJRATISH

V bob. ALGEBRAIK KASRLAR

VI bob. KOMBINATORIKA ELEMENTLARI

22.14
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MIRFAZIL ABDILXAKOVICH MIRZAXMEDOV

ALGEBRA

Umumiy o‘rta ta’lim maktablarining
7- sinfi uchun darslik

Qayta ishlangan va to‘ldirilgan
5- n a s h r i

„O‘qituvchi“ nashriyot-matbaa ijodiy uyi
Toshkent — 2017

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Ijaraga beriladigan darslik holatini ko‘rsatuvchi jadval

T/r	O‘quvchining ismi va familiyasi	O‘quv yili	Darslikning olingandagi holati	Sinf rahbari-ning imzosi	Darslikning topshiril-gandagi holati	Sinf rahbari-ning imzosi
1						
2						
3						
4						
5						

Darslik ijaraga berilib, o‘quv yili yakunida qaytarib olinganda yuqoridagi jadval sinf rahbarlari tomonidan quyidagi baholash mezonlariga asosan to‘ldiriladi:

Yangi	Darslikning birinchi marotaba foydalanishga berilgandagi holati.
Yaxshi	Muqova butun, darslikning asosiy qismidan ajralmagan. Barcha varaqlari mavjud, yirtilmagan, ko‘chmagan, betlarida yozuv va chiziqlar yo‘q.
Qoniqarli	Muqova ezilgan, birmuncha chizilib, chetlari yedirilgan, darslikning asosiy qismidan ajralish holati bor, foydalanuvchi tomonidan qoniqarli ta’mirlangan. Ko‘chgan varaqlari qayta ta’mirlangan, ayrim betlariga chizilgan.
Qoniqarsiz	Muqova chizilgan, yirtilgan, asosiy qismidan ajralgan yoki butunlay yo‘q, qoniqarsiz ta’mirlangan. Betlari yirtilgan, varaqlari yetishmaydi, chizib, bo‘yab tashlangan. Darslikni tiklab bo‘lmaydi.